Group Quiz 3

1. $\vec{r}(t) = \langle e^{2t}, 1 + e^{2t}\cos t, 3 + e^{2t}\sin t \rangle$ @ $(1,2,3)$ or $t=0$.

<u>1st:</u> Find $\vec{T}(t)$.

Let $c = \cos t$ and $s = \sin t$

$$\Rightarrow \vec{r}(t) = \langle e^{2t}, 1 + e^{2t}c, 3 + e^{2t}s \rangle$$

$$\Rightarrow \vec{r}'(t) = \langle 2e^{2t}, 2e^{2t}c - e^{2t}s, 2e^{2t}s + e^{2t}c \rangle$$

$$= e^{2t}\langle 2, 2c - s, 2s + c \rangle$$

$$\Rightarrow |\vec{r}'(t)| = e^{2t}\sqrt{4 + (2c-s)^2 + (2s+c)^2}$$

$$= e^{2t}\sqrt{4 + 4c^2 - \cancel{4cs} + s^2 + 4s^2 + \cancel{4cs} + c^2}$$

$$= 3e^{2t}$$

$$\Rightarrow \vec{T}(t) = \frac{1}{3}\langle 2, 2c - s, 2s + c \rangle \Big|_{t=0} \langle \tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3} \rangle$$

<u>2nd:</u> Find $\vec{N}(0)$

$$\vec{T}'(t) = \frac{1}{3}\langle 0, -2s - c, 2c - s \rangle \Big|_{t=0} \langle 0, -\tfrac{1}{3}, \tfrac{2}{3} \rangle$$

$$\Rightarrow |\vec{T}'(0)| = \sqrt{(\tfrac{1}{3})^2 + (\tfrac{2}{3})^2} = \sqrt{\tfrac{5}{9}} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \vec{N}(0) = \frac{3}{\sqrt{5}}\langle 0, -\tfrac{1}{3}, \tfrac{2}{3} \rangle$$

$$= \langle 0, -\tfrac{1}{\sqrt{5}}, \tfrac{2}{\sqrt{5}} \rangle$$

<u>3rd:</u> Find $\vec{B}(0)$

$$\vec{B}(0) = \vec{T}(0) \times \vec{N}(0)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{vmatrix}$$

$$= \langle \frac{5}{\text{[illegible]}}, -\frac{4}{\text{[illegible]}}, -\frac{2}{\text{[illegible]}} \rangle$$

2 Find a_T and a_N s.t. $\vec{a}(0) = a_T\vec{T}(0) + a_N\vec{N}(0)$

1st: Find $\vec{r}'$, $\vec{r}''$, and $|\vec{r}'|$ @ $t=0$

$$\vec{r}'(t) = e^{2t}\langle 2, 2c-s, 2s+c\rangle\Big|_{t=0} \; 1\cdot\langle 2, 2, 1\rangle$$

$$\vec{r}''(t) = 2e^{2t}\langle 2, 2c-s, 2s+c\rangle + e^{2t}\langle 0, -2s-c, 2c-s\rangle$$

$$= e^{2t}\langle 4, 3c-4s, 3s+4c\rangle\Big|_{t=0} \; 1\cdot\langle 4, 3, 4\rangle$$

$|\vec{r}'(0)| = 3$

also recall $\vec{T}(0) = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$ and $\vec{N}(0) = \left\langle 0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$

2nd: Find a_T and a_N

$$a_T = \frac{\vec{r}'\cdot\vec{r}''}{|\vec{r}'|} \text{ @ } t=0 = \frac{\langle 2,2,1\rangle\cdot\langle 4,3,4\rangle}{3} = \frac{18}{3} = 6$$

$$a_N = \frac{|\vec{r}'\times\vec{r}''|}{|r'|} \text{ @ } t=0 = \frac{\left\|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{vmatrix}\right\|}{3} = \frac{|\langle 5, -4, -2\rangle|}{3} = \frac{\sqrt{45}}{3} = \sqrt{5}$$

3rd: check

$\vec{a} = a_T\vec{T} + a_N\vec{N}$ @ $t=0$

$$\Rightarrow \langle 4,3,4\rangle = 6\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle + \sqrt{5}\left\langle 0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle \checkmark$$

4th: Find the angles.

$\vec{T} \perp \vec{N}$.

To find the angle between $\vec{v}$ and $\vec{a}$ @ $t=0$, use

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|}\right)$$

$$= \cos^{-1}\left(\frac{18}{3\cdot\sqrt{41}}\right)$$

$$= \cos^{-1}\left(\frac{6}{\sqrt{41}}\right) \quad 0.357 \text{ rad or } 20.4^{\circ}$$

3 Find the osculating plane and circle.

1st: Find the curvature when $t=0$.

$$k = \frac{|\vec{T}'(0)|}{|\vec{r}'(0)|} = \frac{\frac{\sqrt{5}}{3}}{3} = \frac{\sqrt{5}}{9}$$

2nd: Osculating plane.

$$\frac{\sqrt{5}}{3}(x-1) - \frac{4}{3\sqrt{5}}(y-2) - \frac{2}{3\sqrt{5}}(z-3) = 0$$

3rd: Osculating circle.

$$\text{kiss}(\theta) = \vec{r}(0) + \frac{1}{k}\vec{N}(0) + \frac{1}{k}\left(\cos\theta\,\vec{T}(0) + \sin\theta\,\vec{N}(0)\right)$$

$$= \langle 1,2,3\rangle + \frac{9}{\sqrt{5}}\left\langle 0,-\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right\rangle + \frac{9}{\sqrt{5}}\left(\cos\theta\left\langle\frac{2}{3},\frac{2}{3},\frac{1}{3}\right\rangle + \sin\theta\left\langle 0,-\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right\rangle\right)$$

$$= \left\langle 1,\frac{1}{5},\frac{33}{5}\right\rangle + \frac{9}{\sqrt{5}}\left\langle\frac{2}{3},\frac{2}{3},\frac{1}{3}\right\rangle\cos\theta + \langle 0,-9,18\rangle\sin\theta$$