high = 91.9%

$\bar{x}$ = 61.8%

med = 59.4%

90s	80s	70s	60s	50s	<50
2	3	4	5	9	6

Test III
Dusty Wilson
Math 153

Name: Key

No work = no credit

No Symbolic Calculators

Combinatorial analysis, in the trivial sense of manipulating binomial and multinomial coefficients, and formally expanding powers of infinite series by applications ad libitum and ad nauseamque of the multinomial theorem, represented the best that academic mathematics could do in the Germany of the late 18th century."

Richard A. Askey (1933 -)
American mathematician

Warm-ups (1 pt each): $\sum_{n=1}^{\infty} 4\cdot\left(\frac{1}{2}\right)^{n-1} = 4\frac{1}{1-\frac{1}{2}} = 8$ $\quad\sum_{n=1}^{\infty}\frac{1}{n} = \infty$ $\quad\lim_{n\to\infty}\frac{5n^2-2}{3n^2+1} = \frac{5}{3}$

1.) (1 pt) How much respect did Askey have for those working on infinite series in the 18th century? Answer using complete English sentences.

Given that he labeled their work as trivial, he didn't have much respect.

ad libitum: for ones pleasure.

ad nauseamque: to ∞ or to nausea.

2.) (10 pts) Does $\sum_{n=1}^{\infty}\frac{10n^3+1}{n^2(n-1)(n-2)}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

L.C.T.

$$\lim_{n\to\infty}\frac{\frac{10n^3+1}{n^2(n-1)(n-2)}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{10n^3+1}{n^2(n-1)(n-2)}\cdot\frac{n}{1}$$

$$= 1$$

since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, we know $\sum_{n=1}^{\infty}\frac{10n^3+1}{n^2(n-1)(n-2)}$ diverges by the L.C.T.

$\frac{3}{10}$ if ratio test & not incl.

$\frac{5}{10}$ if ratio test inconclusive.

T4

74

3.) (10 pts) Does $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^4}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

Integral Test

$\int_2^{\infty} \frac{dx}{x(\ln x)^4}$ Let $u = \ln x$, $du = \frac{dx}{x}$

$= \int_{\ln 2}^{\infty} \frac{du}{u^4}$

$= \lim_{t \to \infty} \int_{\ln 2}^{t} u^{-4}\,du$

$= \lim_{t \to \infty} \left[-\frac{1}{3u^3} \right]_{\ln 2}^{t}$

$= \lim_{t \to \infty} \left(\cancel{-\frac{1}{3t^3}}^{\,0} + \frac{1}{3(\ln 2)^3} \right)$

$= \frac{1}{3(\ln 2)^3}$

since the integral exist/converges we know $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$ converges by the integral test.

4.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{\sqrt[3]{n^4+2}}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

$\sum_{n=1}^{\infty} \frac{\sin^2 n}{\sqrt[3]{n^4+2}} < \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ which is a convergent p-series.

Hence $\sum_{n=1}^{\infty} \frac{\sin^2 n}{\sqrt[3]{n^4+2}}$ converges by the comparison test.

$\overline{20}$

5.) (10 pts) Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{4^n}$ converges. Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} \frac{(x-5)^n}{4^n} = 1 + \frac{x-5}{4} + \frac{(x-5)^2}{4^2} + \ldots$$

$$= \frac{1}{1 - \frac{x-5}{4}} \quad \text{when } \left|\frac{x-5}{4}\right| < 1$$

$$= \frac{4}{9-x} \quad \text{when } -1 < \frac{x-5}{4} < 1$$

$$= \frac{4}{9-x} \quad \text{when } -4 < x-5 < 4$$

$$= \frac{4}{9-x} \quad \text{when. } 1 < x < 9$$

$$= \frac{4}{9-x}$$

6.) (10 pts) Does $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^4}{4^n}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

ratio test

$$\lim_{n \to \infty} \left| \frac{(-1)^{n+2} \frac{(n+1)^4}{4^{n+1}}}{(-1)^{n+1} \frac{n^4}{4^n}} \right|$$

$$= \lim_{n \to \infty} \frac{(n+1)^4}{4 n^4}$$

$$= \frac{1}{4} < 1$$

Hence the series converges by the ratio test.

$\overline{20}$

7.) (10 pts) Determine whether the sequence $a_n = \left(1+\frac{3}{n}\right)^{-n}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{-n} \to 1^{-\infty}$$

$$= e^{\lim_{x\to\infty} \ln\left[\left(1+\frac{3}{x}\right)^{-x}\right]}$$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{-\frac{1}{x}}} \to \frac{0}{0}$$

(H)

$$= e^{\lim_{x\to\infty} \frac{\frac{1}{1+\frac{3}{x}} \cdot -\frac{3}{x^2}}{\frac{1}{x^2}}}$$

$$= e^{\lim_{x\to\infty} \frac{-3}{1+\frac{3}{x}}}$$

$$= e^{-3}$$

8.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^n n!}{2^n (n+1)!}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^n n!}{2^n (n+1)!} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 3^n}{2^n (n+1)}$$

root test.

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^{n+1} 3^n}{2^n (n+1)}\right|} = \lim_{n\to\infty} \frac{3}{2(n+1)^{1/n}}$$

$$= \frac{3}{2} \lim_{x\to\infty} (x+1)^{-1/x}$$

$$= \frac{3}{2} e^{\lim_{x\to\infty} \ln\left[(x+1)^{-1/x}\right]}$$

$$= \frac{3}{2} e^{\lim_{x\to\infty} -\frac{\ln(x+1)}{x}} \to \frac{-\infty}{\infty}$$

(H)

$$= \frac{3}{2} e^{\lim_{x\to\infty} \frac{\frac{1}{x+1}}{1}}$$

$$= \frac{3}{2} > 1$$

so the series diverges by the root test.

20

-90
-00

Test 3
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: Key

Combinatorial analysis, in the trivial sense of manipulating binomial and multinomial coefficients, and formally expanding powers of infinite series by applications ad libitum and ad nauseamque of the multinomial theorem, represented the best that academic mathematics could do in the Germany of the late 18th century."

Richard A. Askey (1933 -)
American mathematician

Warm-ups (1 pt each): $\sum_{n=1}^{\infty} 3\cdot\left(\frac{1}{2}\right)^{n-1} = \frac{3}{1-\frac{1}{2}} = 6$ $\quad\sum_{n=1}^{\infty}\frac{1}{n} = \infty$ $\quad\lim_{n\to\infty}\frac{3n^2-2}{5n^2+1} = \frac{3}{5}$

1.) (1 pt) How much respect did Askey have for those working on infinite series in the 18th century? Answer using complete English sentences.

Not much... he labeled their work as trivial and ad nauseamque.

2.) (10 pts) Does $\sum_{n=1}^{\infty}\frac{n+2^n}{n^2 2^n}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

$$\sum_{n=1}^{\infty}\frac{n+2^n}{n^2 2^n} = \underbrace{\sum_{n=1}^{\infty}\frac{1}{n2^n}}_{\text{convergent by the ratio test. see (*)}} + \underbrace{\sum_{n=1}^{\infty}\frac{1}{n^2}}_{\text{convergent p-series.}}$$

As the sum of convergent series, the series converges.

(*) Ratio Test

$$\lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)2^{n+1}}}{\frac{1}{n2^n}}\right| = \lim_{n\to\infty}\frac{n}{(n+1)\cdot 2} = \frac{1}{2}$$

Hence the series converges by the ratio test

3.) (10 pts) Does $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

$$\int_2^{\infty} \frac{dx}{x(\ln x)^3}$$

Let $u = \ln x$
$du = \frac{1}{x}dx$

$$= \int_{\ln 2}^{\infty} \frac{du}{u^3}$$

$$= \lim_{t\to\infty} \int_{\ln 2}^{t} \frac{du}{u^3}$$

$$= \lim_{t\to\infty} \left[-\frac{1}{2}u^{-2}\right]_{\ln 2}^{t}$$

$$= \lim_{t\to\infty} \left(\overset{0}{\cancel{\frac{-1}{2t^2}}} + \frac{1}{2(\ln 2)^2} \right)$$

$$= \frac{1}{2(\ln 2)^2}$$

Hence $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ converges by the integral test.

4.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{\sqrt[3]{n^4+2}}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{\sin^2 n}{\sqrt[3]{n^4+2}} \leq \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$$ convergent p-series

so the series converges by the comparison test.

5.) (10 pts) Write $3.\overline{14}$ as the ratio of two integers (as a fraction).

$$3.\overline{14} = 3 + \frac{14}{100} + \frac{14}{100^2} + \frac{14}{100^3} + \dots$$

$$= 3 + \frac{\frac{14}{100}}{1 - \frac{14}{100}}$$

$$= 3 + \frac{\frac{14}{100}}{\frac{86}{100}}$$

$$= 3 + \frac{14}{86}$$

$$= 3 + \frac{7}{43}$$

$$= \frac{129 + 7}{43}$$

$$= \frac{136}{43}$$

6.) (10 pts) Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{5^n}$ ~~diverges~~ converges. Find the sum of the series for those values of x.

$$1 + \frac{x-4}{5} + \left(\frac{x-4}{5}\right)^2 + \dots$$

$$= \frac{1}{1 - \frac{x-4}{5}} \quad \text{when } \left|\frac{x-4}{5}\right| < 1$$

$$\Rightarrow -1 < \frac{x-4}{5} < 1$$

$$= \frac{1}{\frac{5 - x + 4}{5}}$$

$$\Rightarrow -5 < x - 4 < 5$$

$$= \frac{5}{9 - x}$$

$$\Rightarrow -1 < x < 9$$

7.) (10 pts) Does $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3}{3^n}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

if root test, must work thru $\lim_{n\to\infty} n^{3/n}$ ind. form, else 8/10

ratio test

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2} \frac{(n+1)^3}{3^{n+1}}}{(-1)^{n+1} \frac{n^3}{3^n}} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)^3 \cdot 3^n}{n^3 \cdot 3^{n+1}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^3}{3n^3} = \frac{1}{3}$$

so the series converges absolutely by the ratio test.

8.) (10 pts) Determine whether the sequence $a_n = \left(1 - \frac{4}{n}\right)^n$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \left(1 - \frac{4}{n}\right)^n \to 1^\infty$$

$$\lim_{x\to\infty} \ln\left[\left(1 - \frac{4}{x}\right)^x\right]$$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1-\frac{4}{x}\right)}{\frac{1}{x}}} \to \frac{0}{0}$$

$$= e$$

$$\overset{H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1-\frac{4}{x}} \cdot \frac{4}{\cancel{x^2}}}{-\frac{1}{\cancel{x^2}}}}$$

$$= e^{\lim_{x\to\infty} \frac{-4}{1-4/x}}$$

$$= e^{-4}.$$

9.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^n (n+1)!}{3^n n!}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

Ratio test.

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+2} 2^{n+1} (n+2)!}{3^{n+1} (n+1)!}}{\frac{(-1)^{n+1} 2^n (n+1)!}{3^n n!}} \right|$$

$$= \lim_{n\to\infty} \frac{2\,(n+2)}{3\,(n+1)}$$

$$= \frac{2}{3} < 1$$

Hence the series converges by the ratio test.