high = 98%

$\bar{x}$ = 72.8%

med = 75%

s_x = 12.9%

Test 1 (Version π)
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: key.

I myself, a professional mathematician, on re-reading my own work find it strains my mental powers to recall to mind from the figures the meanings of the demonstrations, meanings which I myself originally put into the figures and the text from my mind. But when I attempt to remedy the obscurity of the material by putting in extra words, I see myself falling into the opposite fault of becoming chatty in something mathematical.

Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{i} \cdot \vec{i} =$ 1 $\quad \vec{j} \times \vec{k} =$ $\vec{i}$ $\quad \vec{i} - \vec{k} =$ $\langle 1, 0, -1 \rangle$

1.) (1 pt) Based upon the quote above, how easily did Kepler understand his earlier work? Answer using complete English sentences.

kepler struggled when re-reading his own writing

2.) (12 pts) Consider $\vec{u} = \langle 1,2,3 \rangle$ and $\vec{v} = \langle 3,1,4 \rangle$.

a.) Find $\vec{u} - 2\vec{v}$

$= \langle 1, 2, 3 \rangle - \langle 6, 2, 8 \rangle$

$= \langle -5, 0, -5 \rangle$

b.) Find $\vec{u} \cdot \vec{v}$

$= \langle 1, 2, 3 \rangle \cdot \langle 3, 1, 4 \rangle$

$= 3 + 2 + 12$

$= 17$

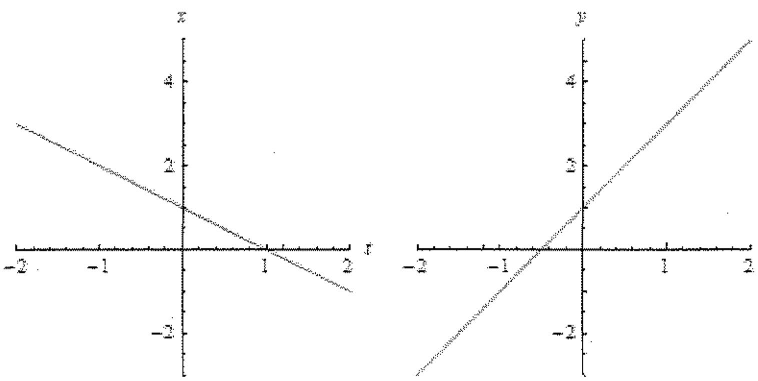
c.) Find $\vec{u} \times \vec{v}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 3 & 1 & 4 \end{vmatrix}$$

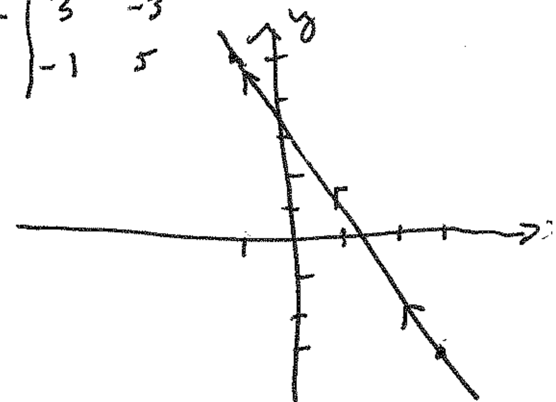
$= \langle 5, 5, -5 \rangle$

$\overline{16}$ $\quad \overline{80}$

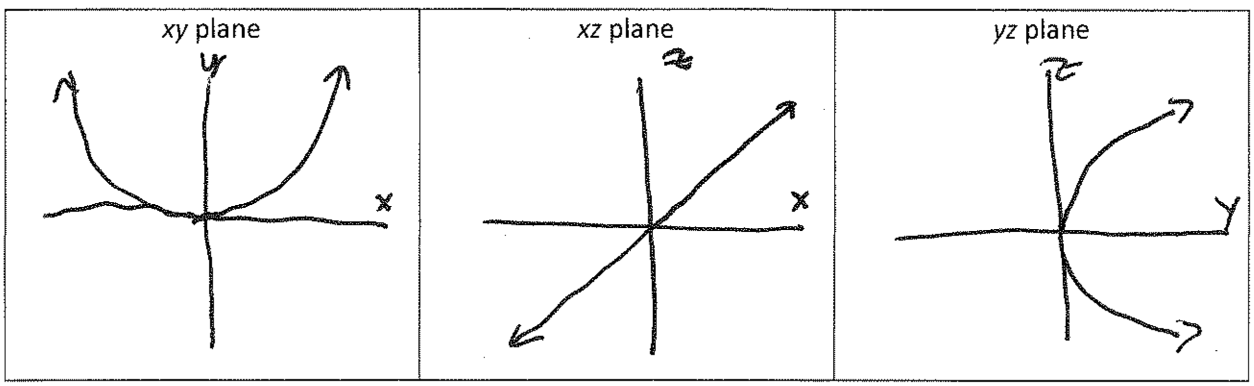
3.) (10 pts) Use the graphs of $x = f(t)$ and $y = g(t)$ to carefully sketch the parametric curve $x = f(t), y = g(t)$. Indicate with arrows the direction which the curve is traced as t increases.



t	x	y
-2	3	-3
2	-1	5



4.) (12 pts) Draw the projections of the curve $\vec{r}(t) = \langle t, t^2, t \rangle$ on the three coordinate planes.



5.) (12 pts) Consider the plane $x + 2y + 3z = 4$

a.) Find the distance from the plane to the point $A(3,1,4)$.

$$D = \frac{|1\cdot 3 + 2\cdot 1 + 3\cdot 4 - 4|}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{13}{\sqrt{14}}$$

b.) Find the equation of the line that is normal to the plane through point A. Give your answer parametrically.

$$\vec{r}(t) = \langle 3, 1, 4 \rangle + t\langle 1, 2, 3 \rangle$$
$$= \langle 3 + t, 1 + 2t, 4 + 3t \rangle$$

34

6.) (10 pts) Find an equation of the plane that passes through the point $(-1, 1, 2)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 4$

<u>1st</u>: Find the line.

$y=0$
$x-z=2$
$2x+3z=4$
$5x=10 \Rightarrow x=2$
$z=0$
$(2,0,0)$

pt: If $x=0$

$$\left.\begin{aligned} y - z &= 2 \\ -y + 3z &= 4 \end{aligned}\right\} \; 2z = 6 \Rightarrow z = 3, \quad y = 5$$

pt $(0,5,3)$.

<u>direction</u>

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = \langle 2, -5, -3 \rangle$$

This vector is <u>on</u> the plane.

<u>2nd</u>: Find a second vector on the plane.

$$\langle 0,5,3 \rangle - \langle -1,1,2 \rangle = \langle 1,4,1 \rangle$$

<u>3rd</u>: Find normal.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -5 & -3 \\ 1 & 4 & 1 \end{vmatrix} = \langle 7, -5, 13 \rangle$$

<u>soln</u>: $7(x+1) - 5(y-1) + 13(z-2) = 0.$

7.) (10 pts) Consider the equation $4x^2 + y^2 + 4z^2 - 40z + 96 = 0$. Reduce the equation to one of the standard forms. Classify the surface and give its center.

$$4x^2 + y^2 + 4z^2 - 40z + 96 = 0$$

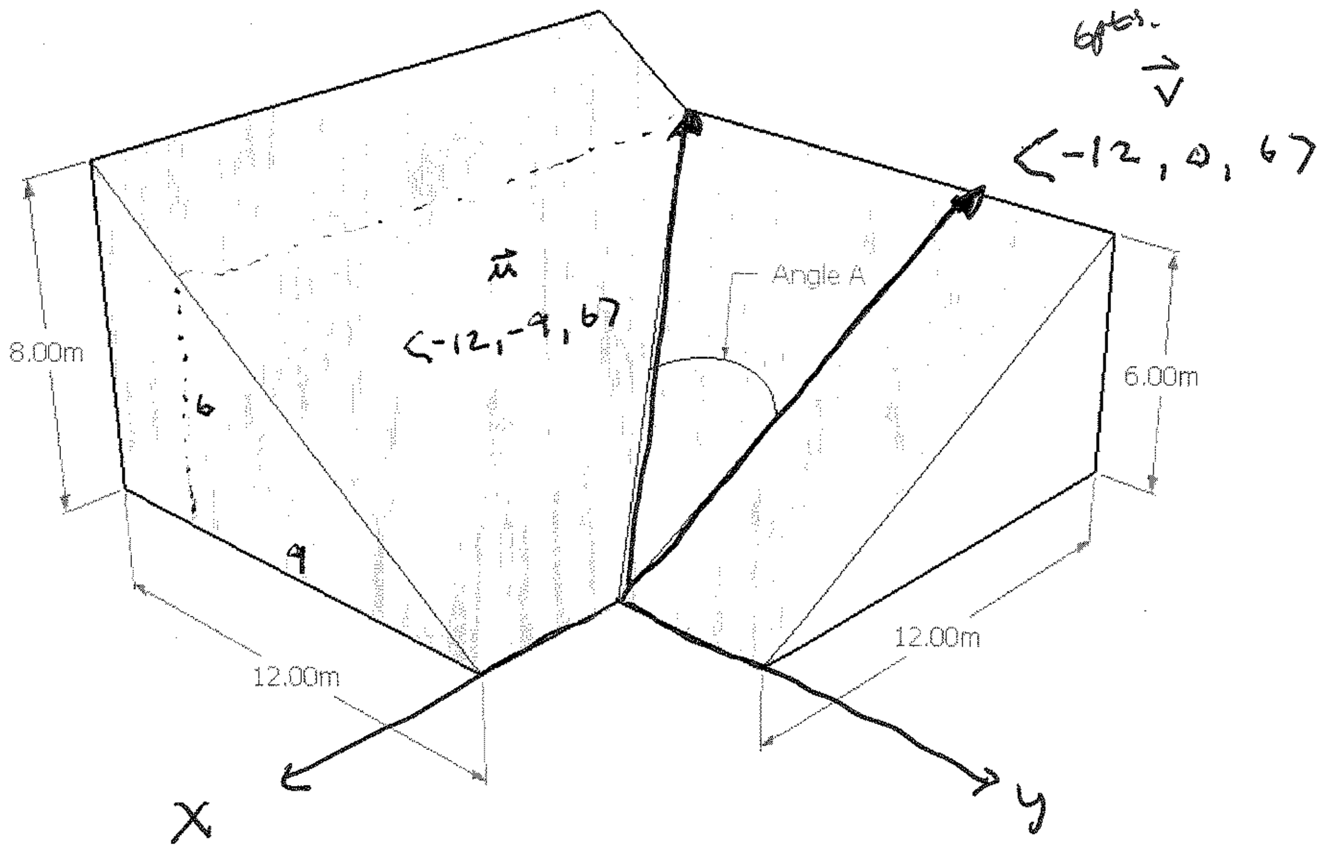
$$\Rightarrow x^2 + \frac{y^2}{4} + z^2 - 10z + 25 = -24 + 25$$

$$\Rightarrow x^2 + \frac{y^2}{4} + (z-5)^2 = 1$$

ellipsoid centered @ $(0,0,5)$.

$\overline{20}$

8.) (10 pts) Determine Angle A (you may give your answer in either degrees or radians).



$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$$

$$\Rightarrow 180 = \sqrt{261}\sqrt{180} \cos \theta$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{180}{\sqrt{261 \cdot 180}}\right)$$

$$= 33.9^{\circ} \quad \text{OR} \quad 0.59 \text{ rad.}$$

$\overline{10}$

Test 1 (Version *e*)
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: Key.

I myself, a professional mathematician, on re-reading my own work find it strains my mental powers to recall to mind from the figures the meanings of the demonstrations, meanings which I myself originally put into the figures and the text from my mind. But when I attempt to remedy the obscurity of the material by putting in extra words, I see myself falling into the opposite fault of becoming chatty in something mathematical.

Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{i}\cdot\vec{k}=$ 0 $\qquad \vec{j}\times\vec{i}=$ $-\vec{k}$ $\qquad \vec{k}-\vec{j}=$ $\langle 0,-1,1\rangle$

1.) (1 pt) Based upon the quote above, how easily did Kepler understand his earlier work? Answer using complete English sentences.

It was tough for kepler to read his own work.

2.) (12 pts) Consider $\vec{u}=\langle 1,2,3\rangle$ and $\vec{v}=\langle 2,7,1\rangle$.

a.) Find $\vec{u}-2\vec{v}$

$= \langle 1,2,3\rangle - \langle 4,14,2\rangle = \langle -3,-12,1\rangle$

b.) Find $\vec{u}\cdot\vec{v}$

$= 1\cdot 2 + 2\cdot 7 + 3\cdot 1$

$= 19$

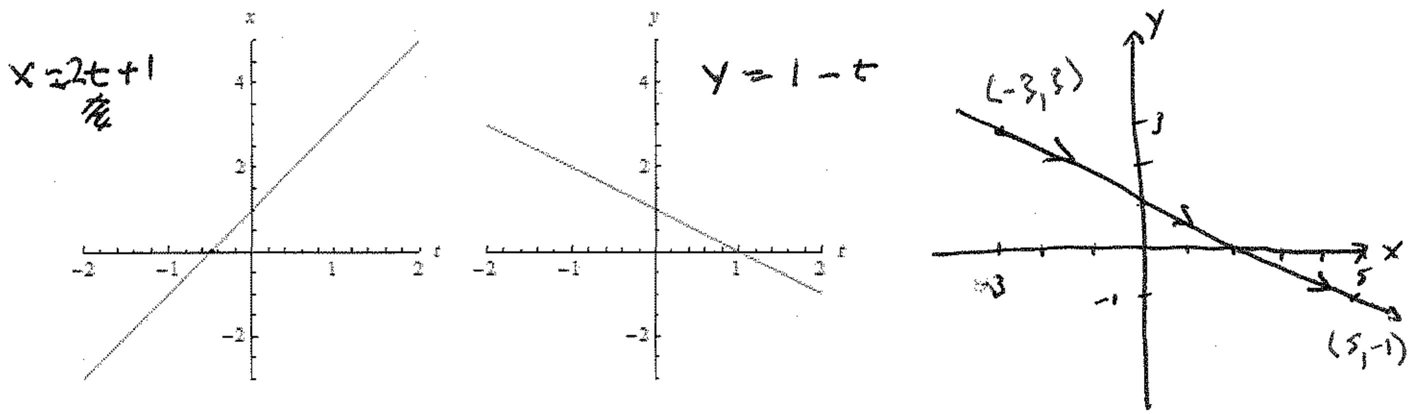
c.) Find $\vec{u}\times\vec{v}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & 7 & 1 \end{vmatrix}$$

$= \langle -19, 5, 3\rangle$

16 $\qquad$ 80

3.) (10 pts) Use the graphs of $x = f(t)$ and $y = g(t)$ to carefully sketch the parametric curve $x = f(t), y = g(t)$. Indicate with arrows the direction which the curve is traced as t increases.



4.) (12 pts) Consider the plane $x + 2y + 3z = 4$

a.) Find the distance from the plane to the point $A(4,5,6)$.

$$D = \frac{|1\cdot 4 + 2\cdot 5 + 3\cdot 6 - 4|}{\sqrt{1^2 + 2^2 + 3^2}}$$

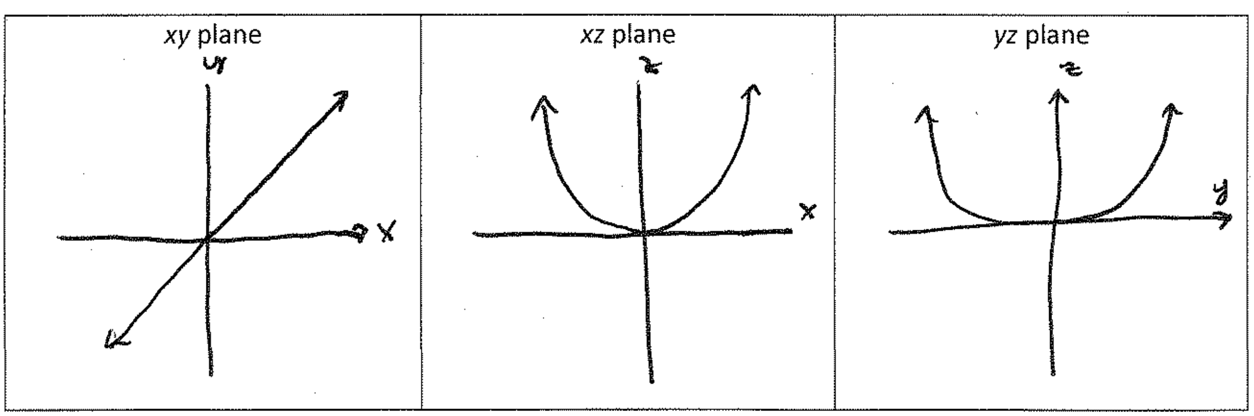
$$= \frac{28}{\sqrt{14}}$$

b.) Find the equation of the line that is normal to the plane through point A. Give your answer parametrically.

$$\vec{r}(t) = \langle 4, 5, 6\rangle + t\langle 1, 2, 3\rangle$$

$$= \langle 4 + t, 5 + 2t, 6 + 3t\rangle$$

5.) (12 pts) Draw the projections of the curve $\vec{r}(t) = \langle t, t, t^2\rangle$ on the three coordinate planes.



34

6.) (10 pts) Find an equation of the plane that passes through the point $(-1, 2, 2)$ and contains the line of intersection of the planes $x + y - z = 6$ and $2x - y + 3z = 4$

1st: Find the line

pt let $x=0$.

$$\left.\begin{array}{l} y - z = 6 \\ -y + 3z = 4 \end{array}\right\} 2z = 10 \Rightarrow z = 5$$

and $y = 11$

$(0, 11, 5)$

direction: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix}$

$= \langle 2, -5, -3 \rangle$

This vector is on the plane.

2nd: Find a 2nd vector on the plane.

$\langle 0, 11, 5\rangle - \langle -1, 2, 2\rangle$

$= \langle 1, 9, 3 \rangle$

3rd: Find Normal.

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -5 & -3 \\ 1 & 9 & 3 \end{vmatrix}$

$= \langle 12, -9, 23 \rangle$

soln: $12(x+1) - 9(y-2) + 23(z-2) = 0$

7.) (10 pts) Consider the equation $4x^2 + y^2 + 4z^2 - 24z + 32 = 0$. Reduce the equation to one of the standard forms. Classify the surface and give its center.

$4x^2 + y^2 + 4z^2 - 24z + 32 = 0$

$\Rightarrow 4x^2 + y^2 + 4(z^2 - 6z + 9) = -32 + 36$

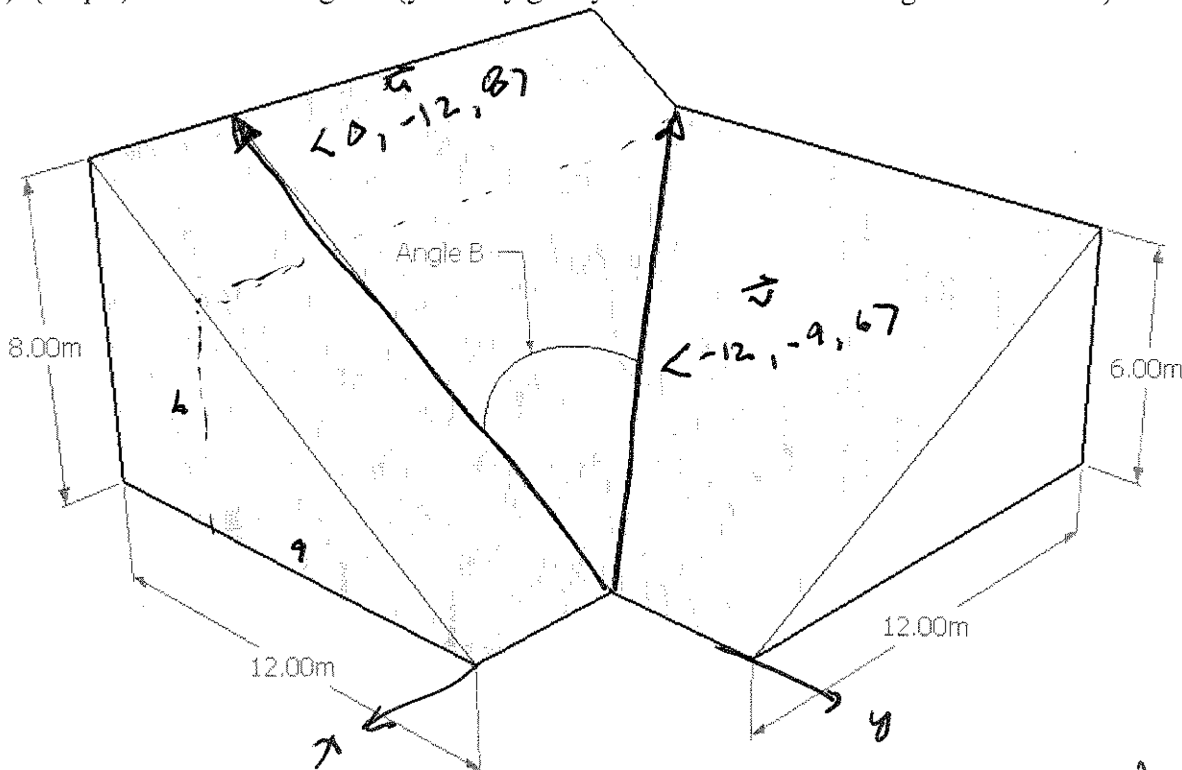
$\Rightarrow 4x^2 + y^2 + 4(z-3)^2 = 4$

$\Rightarrow x^2 + \frac{y^2}{4} + (z-3)^2 = 1$

ellipsoid centered @ $(0, 0, 3)$.

20

8.) (10 pts) Determine Angle B (you may give your answer in either degrees or radians).



$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

$\Rightarrow 156 = \sqrt{208} \cdot \sqrt{261} \cos\theta$

$\Rightarrow \cos\theta = \dfrac{156}{\sqrt{208 \cdot 261}}$

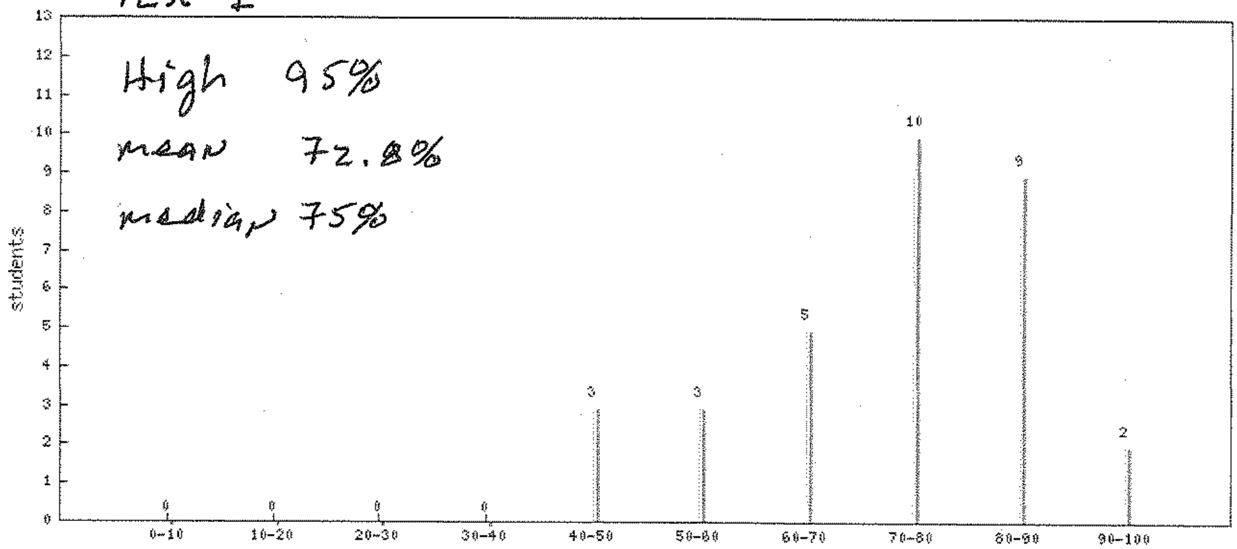
$\theta = 0.837$ rad

47.979°

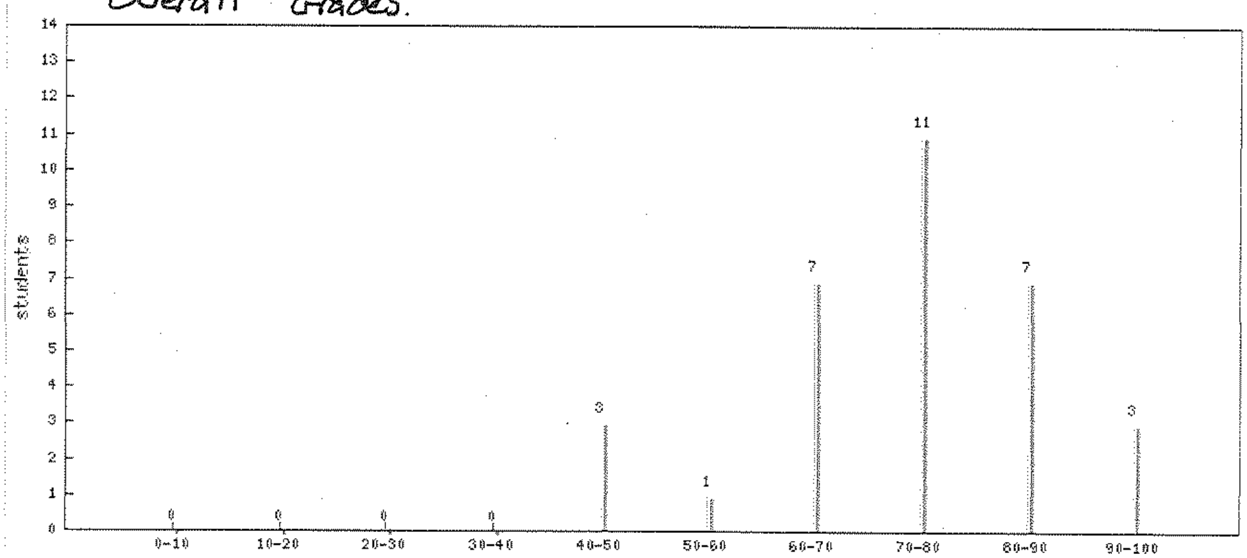
$\frac{6}{10}$ if both $\langle 0, -12, 8 \rangle$ and $\langle -12, 0, 6 \rangle$

TO

Test 1



Overall Grades.



Aim for at least 90% on the HW.

	Below 2.0 in class	At least 2.0 in class
Below 90% on the HW	9	6
At least 90% on the HW	1	16
Total	10	22