Test 2
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: Key.

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

Warm-ups (1 pt each): $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n}\right)=$ 0 $\quad \sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=$ 1 $\quad \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=$ ∞

$(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+\ldots$

1.) (1 pt) According to Euler, what mathematician struggled to understand 1-1+1-1+…? Answer using complete English sentences.

Leibniz had issues w/ divergent sums.

2.) (10 pts) Does $\sum_{n=1}^{\infty}\frac{(-1)^n 3^n}{(2n+1)!}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

ratio test

$$\lim_{n\to\infty}\frac{3^{n+1}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{3^n}=\lim_{n\to\infty}\frac{3}{(2n+3)(2n+2)}=0$$

The sum converges absolutely by the ratio test.

3.) (10 pts) Does $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln(n)}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

A.S.T.

$$\lim_{n\to\infty} \frac{1}{n\ln(n)} = 0$$ conditionally convergent.

Integral Test.

$$\int_2^{\infty} \frac{dx}{x\ln x} = \int_{\ln 2}^{\infty} \frac{du}{u} = \infty \quad (p\text{-test}).$$

so the series converges conditionally by the A.S.T.

4.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+2n+1}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

L.C.T.

$$\lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{1}{n^2+2n+1}} = \lim_{n\to\infty} \frac{n^2+n+1}{n^2} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p series) we know $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+2n+1}$ converges absolutely by the limit comparison test.

5.) (10 pts) Write $3.1\overline{41}$ as the ratio of two integers (as a fraction).

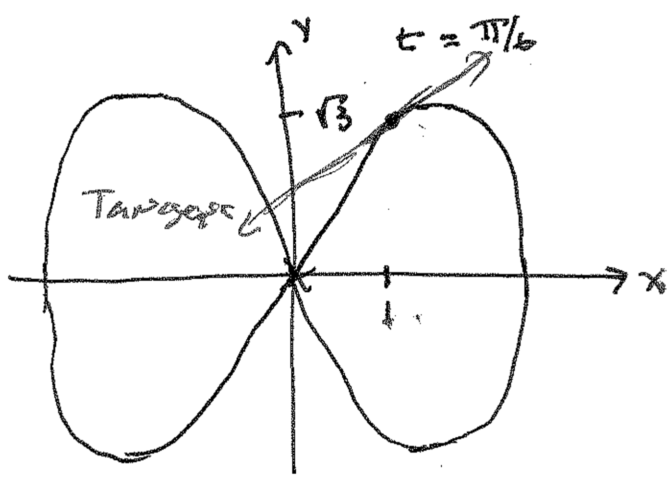
$$3.1\overline{41} = 3.1 + \frac{41}{1000} + \frac{41}{100000} + \cdots$$

$$= 3.1 + \sum_{n=1}^{\infty} \frac{41}{1000} \cdot \left(\frac{1}{100}\right)^{n-1}$$

$$= 3.1 + \frac{41/1000}{1 - \frac{1}{100}}$$

$$= \frac{311}{99}$$

6.) (10 pts) Find the equation of the tangent line to the curve parameterized by $x = 2\sin(2t)$ and $y = 2\sin(t)$ at the point $(\cancel{\sqrt{3},1})$. $(1, \sqrt{3})$



$$x' = 4\cos(2t)$$
$$y' = 2\cos(t)$$
$\Big|_{t=\pi/6}$ $(2, \sqrt{3})$

Tangent

$$y - \sqrt{3} = \frac{\sqrt{3}}{2}(x - 2)$$

or

$$x = 2t + 1 \quad \& \quad y = \sqrt{3}t + \sqrt{3}$$

7.) (10 pts) Set up an integral to represent the length of the curve $x = t^3$ and $y = t^2$ on $0 \le t \le 4$.

Note: You may evaluate the integral to verify the area is $8\left(37^{3/2} - 1\right)/27$

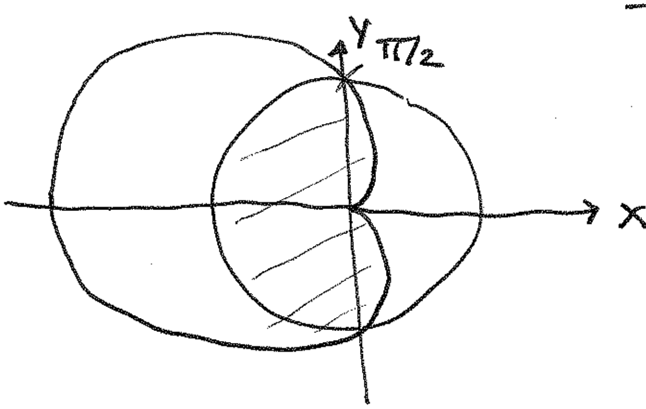
$$S = \int_0^4 \sqrt{(3t^2)^2 + (2t)^2}\, dt$$

$$= \int_0^4 \sqrt{9t^4 + 4t^2}\, dt$$

3pts for radical.

8.) (10 pts) Set up an integral to find the area shared by the circle $r = 2$ and the cardiod $r = 2(1 - \cos\theta)$.

Note: You may evaluate the integral to verify the area is $5\pi - 8$



To find the intersection

$$2 = 2(1 - \cos\theta)$$

$$\Rightarrow 1 = 1 - \cos\theta$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{Area} = \frac{\pi \cdot 2^2}{2} + 2\int_0^{\pi/2} \frac{1}{2}\left[2(1 - \cos\theta)\right]^2 d\theta$$