Group Quiz 3
Dusty Wilson

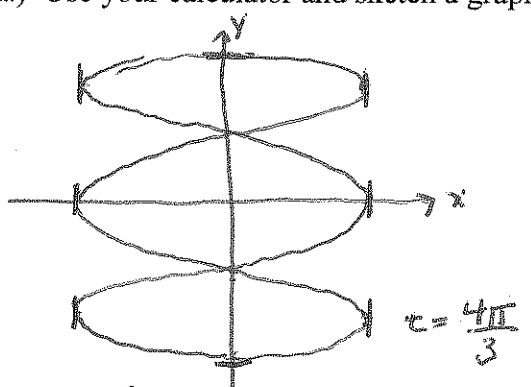
Name: Key

No work = no credit

1.) Consider the parametrically curve:

$$x(t) = 4\cos(3t) \text{ and } y(t) = 4\sin(t)$$

a.) Use your calculator and sketch a graph of the curve.



b.) Find $\frac{dy}{dx}$ (simplification is optional).

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\cos(t)}{-12\sin(3t)} = -\frac{1}{3}\frac{\cos(t)}{\sin(3t)}$$

c.) Set up an equation to find where the tangents are horizontal. Clearly show where these exist on the graph in (a.).

Solve $\cos(t) = 0$

d.) Set up an equation to find where the tangent does not exist. Clearly show where these exist on the graph in (b.). Find the exact coordinates of the point in the fourth quadrant.

Solve $\sin(3t) = 0 \Rightarrow t = \frac{1}{3}\pi k,\ k \in \mathbb{Z}$

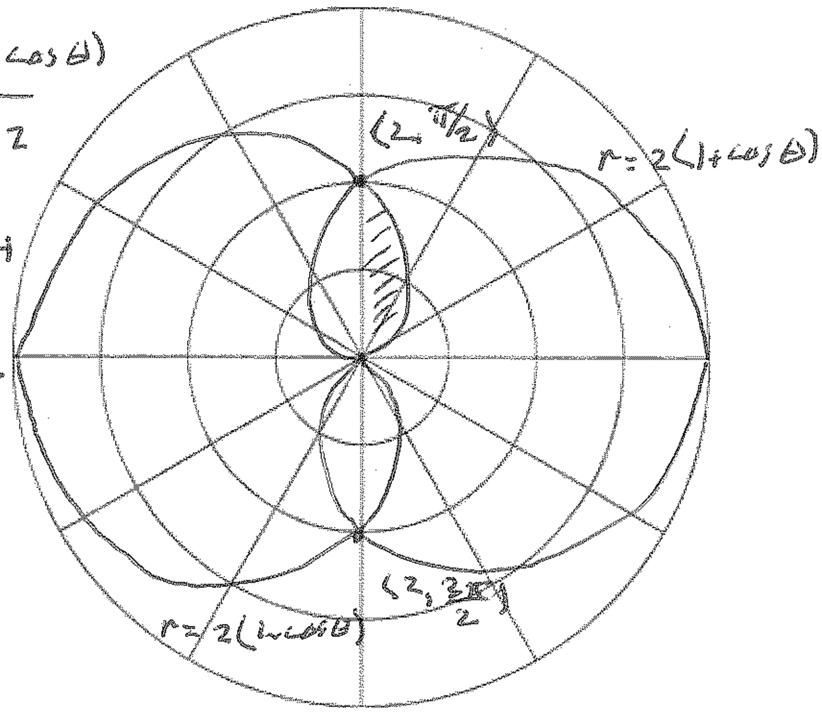
$t = \frac{4\pi}{3}$ $\quad (4, -2\sqrt{3})$

e.) Set up (do not solve) an integral to represent the arclength of the figure.

$$s = \int_0^{2\pi} \sqrt{(-12\sin 3t)^2 + 16\cos^2 t}\, dt$$

2.) (No calculator) Carefully sketch a graph that includes $r = 2(1+\cos(\theta))$ and $r = 2(1-\cos(\theta))$. Make sure to label each graph. Find the point(s) of intersection and express the coordinate(s) in polar form. Explain what happens at the pole.

θ	$2(1+\cos\theta)$	$2(1-\cos\theta)$
$0 \to \frac{\pi}{2}$	$4 \to 2$	$0 \to 2$
$\frac{\pi}{2} \to \pi$	$2 \to 0$	$2 \to 4$
$\pi \to \frac{3\pi}{2}$	$0 \to 2$	$4 \to 2$
$\frac{3\pi}{2} \to 2\pi$	$2 \to 4$	$2 \to 0$



3.) Find the area shared by the two cardiods in the previous question.

$$A = 4\int_0^{\pi/2} \frac{1}{2}\left[2(1-\cos\theta)\right]^2 d\theta$$

$$= 8\int_0^{\pi/2} (1 - 2\cos\theta + \cos^2\theta)\, d\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= 8\left[\frac{3}{2}\theta - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$= 8\left(\frac{3}{4}\pi - 2\right)$$

$$= 6\pi - 16$$