11.8 : Power Series

A power series is a fct of the form $\sum_{n=0}^{\infty} c_n x^n$ whose domain is the set of all x's for which the series converges.

NOTE: c_n's are coefficients, x is the variable.

Ex1: $\sum_{n=0}^{\infty} c \cdot x^n = c + cx + cx^2 + \ldots$

This is a geometric series which converges when $|x| < 1$. and $\sum_{n=0}^{\infty} cx^n = \frac{c}{1-x}$; $|x| < 1$.

Generally: the power series centered at $x = a$ is $\sum_{n=0}^{\infty} c_n (x-a)^n$.

NOTE: When $x = a$, we say $(x-a)^0 = 1$.

Ex 2: When does $\sum_{n=0}^{\infty} n!\, x^n$ converge?

Using the ratio test, we have $\lim_{n\to\infty} \frac{(n+1)!\, x^{n+1}}{n!\, x^n}$
$\hookrightarrow = \lim_{n\to\infty} (n+1)x$. This limit has magnitude less than 1 iff $x = 0$. Otherwise the limit diverges. So the series converges iff $x = 0$.

Ex 3: For what values of x does $\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$ converge?

* use the ratio test .... $|x| < 3$.
* check endpoints: $x = 3$ divergent

  $x = -3$ convergent.

since the series converges on $[-3, 3)$, we say the interval of convergence is $[-3, 3)$

Review: What is the interval of convergence in Ex 1 and Ex 2?

Ex 4: Find the interval of convergence of the series $\sum_{n=0} \frac{x^n}{n!}$

Theorem: For a given power series $\sum_{n=0} c_n (x-a)^n$ there are only three possibilities.

i) converges only at $x = a$.

ii) converges for all x.

iii) $\exists R > 0$ s.t. the series converges when $|x - a| < R$ and diverges when $|x - a| > R$. we call R the radius of convergence.

NOTE: Radius of convergence vs. Interval of convergence

Ex 5: Find the RoC and I.o.C. of $\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$

Ex 6: Find the RoC and I.o.C. of $\sum_{n=0}^{\infty} \sqrt{n}(x-1)^n$

Show cungus example of a drum membrane.