high = 97%

mean = 71.5%

median = 75%

< 60	60's	70's	80's	90's
6	4	11	6	3

Test 2 (version 1) **Name:** key

Dusty Wilson

Math 153

No work = no credit

No Symbolic Calculators

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

Warm-ups (1 pt each): $\vec{T}\times\vec{N} = \vec{B}$ $\quad r\sin(\theta) = y$ $\quad r\cos(\theta) = x$

1.) (1 pt) According to Euler, what mathematician struggled to understand 1-1+1-1+…? Answer using complete English sentences.

Leibniz struggled w/ divergent sums.

2.) (10 pts) Write $\vec{a}(2)$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for the position vector-valued function $\vec{r}(t) = t\,\vec{i} + t^2\vec{j} + 3t\,\vec{k}$ at $t = 2$. That is, you need to find a_T and a_N.

$\vec{r}' = \langle 1, 2t, 3\rangle\big|_{t=2}$ $\quad \langle 1, 4, 3\rangle$

$\vec{r}'' = \langle 0, 2, 0\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 3 \\ 0 & 2 & 0 \end{vmatrix} = \langle -6, 0, 2\rangle$$

$$a_T = \frac{\vec{r}'\cdot\vec{r}''}{|\vec{r}'|} = \frac{8}{\sqrt{26}}$$

$$a_N = \frac{|\vec{r}'\times\vec{r}''|}{|\vec{r}'|} = \frac{\sqrt{40}}{\sqrt{26}}$$

$$\Rightarrow \vec{a} = \frac{8}{\sqrt{26}}\vec{T} + \sqrt{\frac{40}{26}}\vec{N}$$ $\leftarrow$ 1 pt.

3.) (10 pts) Find $\vec{T}, \vec{N}$, and $\vec{B}$ for $\vec{r}(t) = \langle 5\cos t, 5\sin t, 12t \rangle$

$$\vec{r}' = \langle -5\sin t, 5\cos t, 12 \rangle$$

$$|\vec{r}'| = \sqrt{25+144} = 13$$

$$\Rightarrow \vec{T}(t) = \frac{1}{13}\langle -5\sin t, 5\cos t, 12 \rangle$$

$$\vec{T}'(t) = \frac{5}{13}\langle -\cos t, -\sin t, 0 \rangle$$

$$|\vec{T}'(t)| = \frac{5}{13}$$

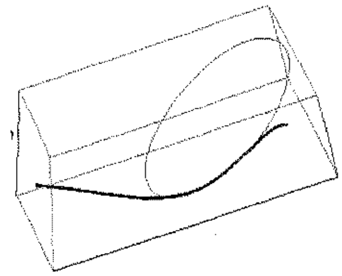
$$\Rightarrow \vec{N}(t) = \langle -\cos t, -\sin t, 0 \rangle$$

$$\vec{T}(t) \times \vec{N}(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{5}{13}\sin t & \frac{5}{13}\cos t & \frac{12}{13} \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$\Rightarrow \vec{B}(t) = \left\langle \frac{12}{13}\sin t, -\frac{12}{13}\cos t, \frac{5}{13} \right\rangle$$

4.) (10 pts) Find the kissing circle of $\vec{r}(t)=\langle t^2, \sin(t)-t\cos(t), \cos(t)+t\sin(t)\rangle$ when $t=\pi$ given that at this t value:

$\Rightarrow \vec{r}(\pi) = \langle \pi^2, +\pi, -1 \rangle$



$$\vec{T}(t)=\frac{1}{\sqrt{5}}\langle 2, \sin(t), \cos(t)\rangle\Big|_{t=\pi} \frac{1}{\sqrt{5}}\langle 2,0,-1\rangle$$

$$\vec{N}(t)=\langle 0, \cos(t), -\sin(t)\rangle\Big|_{t=\pi} \langle 0,-1,0\rangle$$

$$\vec{r}\,'(t)=\langle 2t, t\sin t, t\cos t\rangle\Big|_{t=\pi} \langle 2\pi, 0, -\pi\rangle$$

$$\vec{r}\,''(t)=\langle 2, \sin t + t\cos t, \cos t - t\sin t\rangle\Big|_{t=\pi} \langle 2, -\pi, -1\rangle$$

$$|\vec{r}\,'(t)| = \sqrt{4\pi^2+\pi^2} = \sqrt{5}\pi$$

find the curvature.

$$\langle 2\pi, 0, -\pi\rangle \times \langle 2, -\pi, -1\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2\pi & 0 & -\pi \\ 2 & -\pi & -1 \end{vmatrix}$$

$$= \langle -\pi^2, 0, -2\pi^2\rangle \text{ w/mag } \sqrt{5}\pi^2$$

$$\text{so } \kappa = \frac{\sqrt{5}\pi^2}{(\sqrt{5}\pi)^3} = \frac{1}{5\pi} \text{ and radius} = 5\pi$$

$$\Rightarrow \text{kissing circle} = \langle \pi^2, \pi, -1\rangle + 5\pi\langle 0,-1,0\rangle + 5\pi\cos\theta \cdot \frac{1}{\sqrt{5}}\langle 2,0,-1\rangle + 5\pi\sin\theta\langle 0,-1,0\rangle$$

5.) (10 pts) Find the equation of the tangent line to the curve parameterized by $x=2\cos(t)$ and $y=2\sin(t)$ when $t=\frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos(t)}{-2\sin(t)}\Bigg|_{t=\pi/6} \frac{2\frac{\sqrt{3}}{2}}{-2\frac{1}{2}} = -\sqrt{3}$$

$$\text{pt } \left(2\frac{\sqrt{3}}{2}, 2\cdot\frac{1}{2}\right) = (\sqrt{3}, 1)$$

$$\text{tangent: } y-1 = -\sqrt{3}(x-\sqrt{3})$$

6.) (10 pts) Use techniques developed in this course to verify that the surface area of a sphere with radius 3 is 36π.

Hint: Begin by writing a parametric equation for a circle of radius 3 centered at the origin.

SA of a sphere

$\frac{8}{10}$ if $\int_0^{2\pi} \neq 0.$

$x = r\cos(t)$

$y = r\sin(t)$

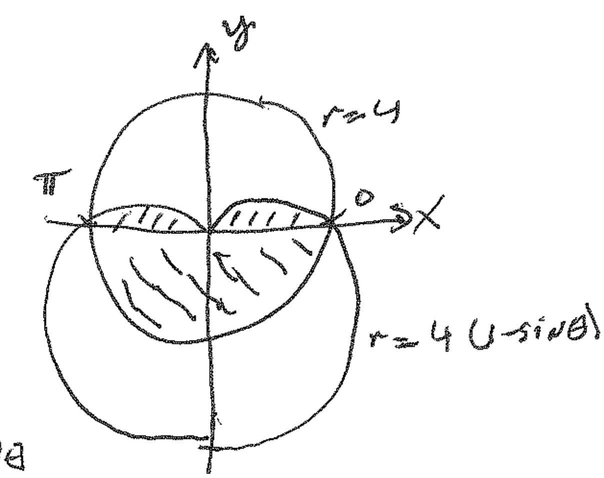
$$SA = \int_0^{\pi} 2\pi r\sin t \sqrt{(r\sin t)^2 + r^2\cos^2 t}\, dt$$

$$= \int_0^{\pi} 2\pi r^2 \sin t\, dt$$

$$= 4\pi r^2 \Big|_{r=3} \quad 36\pi$$

7.) (10 pts) <u>Set up</u> an integral to find the area shared by the circle $r = 4$ and the cardiod $r = 4(1-\sin\theta)$.

Note: You may evaluate the integral to verify the area is $20\pi - 32$



solve $4 = 4(1-\sin\theta)$

$\Rightarrow \quad 1 = 1 - \sin\theta$

$\Rightarrow \quad 0 = \sin\theta$

$\Rightarrow \quad \theta = 0$ OR π

$$\text{Area} = \int_0^{\pi} \frac{1}{2}\left[4(1-\sin\theta)\right]^2 d\theta + \int_{\pi}^{2\pi} \frac{1}{2}(4)^2\, d\theta$$

$$= \int_0^{\pi} 8(1-\sin\theta)^2 d\theta + 8\pi$$

Test II
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: Key.

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

Warm-ups (1 pt each): $\bar{T}\times\bar{N} = \vec{B}$ $\quad r\cos(\theta) = x$ $\quad r\sin(\theta) = y$

1.) (1 pt) According to Euler, what mathematician struggled to understand 1-1+1-1+...? Answer using complete English sentences.

Leibniz struggled to understand.

2.) (10 pts) Write $\bar{a}(3)$ in the form $\bar{a} = a_T\bar{T} + a_N\bar{N}$ without finding $\bar{T}$ and $\bar{N}$ for the position vector-valued function $\bar{r}(t) = t^2\,\bar{i} + t\,\bar{j} + 5t\,\bar{k}$ at $t = 3$. That is, you need to find a_T and a_N.

$$\vec{r}'(t) = \langle 2t, 1, 5\rangle\Big|_{t=3} \quad \langle 6, 1, 5\rangle$$

$$|\vec{r}'(2)| = \sqrt{36+1+25} = \sqrt{62}$$

$$\vec{r}''(t) = \langle 2, 0, 0\rangle$$

$$\vec{r}'(2)\cdot\vec{r}''(2) = 12$$

$$|\vec{r}'(2)\times\vec{r}''(2)| = \text{mag}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 1 & 5 \\ 2 & 0 & 0 \end{vmatrix} = |\langle 0, 10, -2\rangle| = \sqrt{104}$$

$$a_T = \frac{12}{\sqrt{62}} \text{ and } a_{\vec{N}} = \sqrt{\frac{104}{62}}$$

$$\vec{a} = \frac{12}{\sqrt{62}}\vec{T} + \sqrt{\frac{104}{62}}\vec{N}.$$

T4 $\quad$ $\overline{64}$

3.) (10 pts) Find $\vec{T}, \vec{N}$, and $\vec{B}$ for $\vec{r}(t) = \langle 15\cos t, 15\sin t, 8t \rangle$

$$\vec{r}\,' = \langle -15\sin t, 15\cos t, 8 \rangle$$

$$|\vec{r}\,'| = \sqrt{225 + 64} = 17$$

$$\Rightarrow \vec{T}(t) = \frac{1}{17}\langle -15\sin t, 15\cos t, 8 \rangle$$

$$\Rightarrow \vec{T}\,'(t) = \frac{15}{17}\langle -\cos t, -\sin t, 0 \rangle$$

$$|T'(t)| = \frac{15}{17}$$

$$\Rightarrow \vec{N}(t) = \langle -\cos(t), -\sin t, 0 \rangle$$

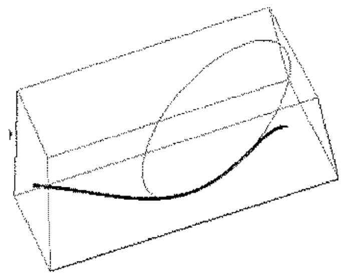
$$\vec{T}(t) \times \vec{N}(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{15}{17}\sin t & \frac{15}{17}\cos t & \frac{8}{17} \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$\Rightarrow \qquad \vec{B}(t) = \left\langle \frac{8}{17}\sin t, -\frac{8}{17}\cos t, \frac{15}{17} \right\rangle$$

$\overline{10}$

4.) (10 pts) Find the kissing circle of $\vec{r}(t) = \langle t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t) \rangle$ when $t = \pi$ given that at this t value: $\Rightarrow \vec{r}(\pi) = \langle \pi^2, +\pi, -1 \rangle$

$$\vec{T}(t) = \frac{1}{\sqrt{5}} \langle 2, \sin(t), \cos(t) \rangle \Big|_{t=\pi} \frac{1}{\sqrt{5}} \langle 2, 0, -1 \rangle$$
$$\vec{N}(t) = \langle 0, \cos(t), -\sin(t) \rangle \Big|_{t=\pi} \langle 0, -1, 0 \rangle$$
$$\vec{r}\,'(t) = \langle 2t, t\sin t, t\cos t \rangle \Big|_{t=\pi} \langle 2\pi, 0, -\pi \rangle$$
$$\vec{r}\,''(t) = \langle 2, \sin t + t\cos t, \cos t - t\sin t \rangle \Big|_{t=\pi} \langle 2, -\pi, -1 \rangle$$



$$|\vec{r}\,'(t)| = \sqrt{4\pi^2 + \pi^2} = \sqrt{5}\pi$$

find the curvature.

$$\langle 2\pi, 0, -\pi \rangle \times \langle 2, -\pi, -1 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\pi & 0 & -\pi \\ 2 & -\pi & -1 \end{vmatrix}$$

$$= \langle -\pi^2, 0, -2\pi^2 \rangle \text{ w/mag } \sqrt{8}\pi^2$$

so $\kappa = \dfrac{\sqrt{5}\pi^2}{(\sqrt{5}\pi)^3} = \dfrac{1}{5\pi}$ and radius $= 5\pi$

$\Rightarrow$ kissing circle $= \langle \pi^2, \pi, -1 \rangle + 5\pi \langle 0, -1, 0 \rangle + 5\pi \cos\theta \cdot \frac{1}{\sqrt{5}} \langle 2, 0, -1 \rangle$

$+ 5\pi \sin\theta \langle 0, -1, 0 \rangle$

5.) (10 pts) Find the equation of the tangent line to the curve parameterized by $x = 2\sin(t)$ and $y = 2\cos(t)$ when $t = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{2\cos t}\Bigg|_{t=\frac{\pi}{3}} \quad \frac{-2\frac{\sqrt{3}}{2}}{2 \cdot \frac{1}{2}} = -\sqrt{3}$$

pt: $\left(2 \cdot \frac{\sqrt{3}}{2}, 2 \cdot \frac{1}{2}\right) = (\sqrt{3}, 1)$

$\frac{6}{10}$ if just unit tangent

tangent: $y - 1 = -\sqrt{3}(x - \sqrt{3})$

$\Rightarrow y = -\sqrt{3}x + 4$

$\overline{20}$

6.) (10 pts) Use techniques developed in this course to verify that the surface area of a sphere with radius 5 is 100π.

Hint: Begin by writing a parametric equation for a circle of radius 5 centered at the origin.

SA of a sphere.

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$SA = \int_0^{\pi} 2\pi r \sin t \sqrt{(-r\sin t)^2 + (r\cos t)^2}\, dt$$

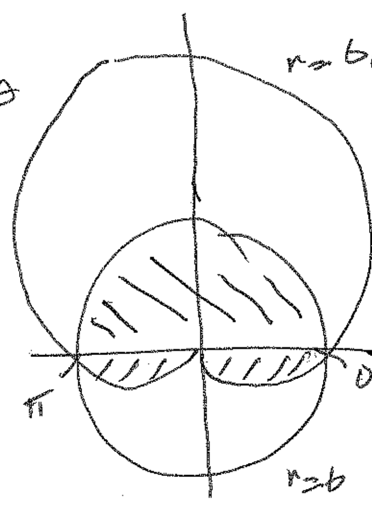
$$= \int_0^{\pi} 2\pi r^2 \sin t \, dt$$

$$= 4\pi r^2 \Big|_{r=5} \qquad 100\pi$$

7.) (10 pts) Set up an integral to find the area shared by the circle $r = 6$ and the cardiod $r = 6(1+\sin\theta)$.

Note: You may evaluate the integral to verify the area is $45\pi - 72$

$$Area = \frac{1}{2}\int_0^{\pi} (6)^2 d\theta + \frac{1}{2}\int_{\pi}^{2\pi} \left[6(1+\sin\theta)\right]^2 d\theta$$



20