Name: key

Homework Quiz 01: 12.2#10

Find a vector that has the same direction as $\langle -6, 4, 2\rangle$ but has length 6.

length: $\sqrt{36+16+4} = \sqrt{56}$

soln. $\frac{6}{\sqrt{56}} \langle -6, 4, 2\rangle$

Name: Key

Homework Quiz 02: 12.3#14

Find the scalar and vector projections of $\vec{b} = \langle 2,3,4 \rangle$ onto $\vec{a} = \langle 5,-6,-1 \rangle$

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{-12}{\sqrt{62}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a} = \frac{-12}{62}\langle 5,-6,-1\rangle$$

$$= -\frac{6}{31}\langle 5,-6,-1\rangle$$

$\text{comp}_{\vec{a}}\vec{b} = -\frac{12}{\sqrt{62}}$

$\text{proj}_{\vec{a}}\vec{b} = \frac{-6}{31}\langle 5,-6,-1\rangle$

Name: Key

Homework Quiz 03: 12.4#9

Consider the points $P(1,0,1)$, $Q(-2,1,4)$, and $R(7,2,7)$.

a.) Find a nonzero vector orthogonal to the plane through the points *P*, *Q*,and *R*.

$\vec{PQ} = \langle -3, 1, +3 \rangle$

$\vec{PR} = \langle 6, 2, 6 \rangle$

$\vec{PQ} \times \vec{PR} = \langle 0, +36, -12 \rangle$

b.) Find the area of the <u>triangle</u> *PQR*.

$$\text{Area} = \frac{|\vec{PQ} \times \vec{PR}|}{2} = \frac{\sqrt{36^2 + 12^2}}{2} = \frac{12\sqrt{10}}{2} = 6\sqrt{10}$$

Name: key.

Homework Quiz 04: 12.5#5

Find parametric equations and symmetric equations for the line through (3, 3, 0) and perpendicular to both **i** + **j** and **j** + **k**

$$\langle 1,1,0\rangle \times \langle 0,1,1\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \langle 1,-1,1\rangle$$

Parametric equations

$$\vec{r}(t) = \langle 3,3,0\rangle + t\langle 1,-1,1\rangle$$

$$= \langle 3+t, 3-t, t\rangle$$

Symmetric equations

$$x-3 = 3-y = z$$

Name: Key

Homework Quiz 05: 12.5#11

Find an equation of the plane through the points $\overset{A}{(0, 4, 4)}$, $\overset{B}{(4, 0, 4)}$, and $\overset{C}{(4, 4, 0)}$

$\overrightarrow{AB} = \langle 4, -4, 0 \rangle$

$\overrightarrow{AC} = \langle 4, 0, -4 \rangle$

$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 16, 16, 16 \rangle$

plane $16(x-0) + 16(y-4) + 16(z-4) = 0$.

$16x + 16y + 16z = 128$

Name: Key.

Homework Quiz 06: 12.5#17

2nd version on the next page

Find an equation of the plane that passes through the point $(-2, 3, 2)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 3$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = \langle 2, -5, -3 \rangle$$ Direction of the line of intersection.

I need one point on the line of intersection.

pick $y=0$: $\begin{matrix} x - z = 2 \\ 2x + 3z = 3 \end{matrix} \Rightarrow x = \frac{9}{5}$ and $z = -\frac{1}{5}$

so $\left(\frac{9}{5}, 0, -\frac{1}{5}\right)$ is one pt. of intersection.

$\Rightarrow \langle -2, 3, 2 \rangle - \langle \frac{9}{5}, 0, -\frac{1}{5} \rangle = \langle -\frac{19}{5}, 3, \frac{11}{5} \rangle$ is a vector // to the plane

$$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -5 & -3 \\ -\frac{19}{5} & 3 & \frac{11}{5} \end{vmatrix} = \langle -2, 7, -13 \rangle$$ is $\perp$ to the plane

$-2(x+2) + 7(y-3) - 13(z-2) = 0$

$\Rightarrow -2x + 7y - 13z = -1$

Quiz 6: A second version.

Planes: $x+y-z=4$ and $4x-y+5z=4$

A direction vector for the line of intersection is $\mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, -1\rangle \times \langle 4, -1, 5\rangle = \langle 4, -9, -5\rangle$, and $\mathbf{a}$ is parallel to the desired plane. Another vector parallel to the plane is the vector connecting any point on the line of intersection to the given point $(-2, 2, 3)$ in the plane. Setting $x = 0$, the equations of the planes reduce to $y - z = 4$ and $-y + 5z = 4$ with simultaneous solution $y = 6$ and $z = 2$. So a point on the line is $(0, 6, 2)$ and another vector parallel to the plane is $\langle -2, -4, 1\rangle$. Then a normal vector to the plane is $\mathbf{n} = \langle 4, -9, -5\rangle \times \langle -2, -4, 1\rangle = \langle -29, 6, -34\rangle$ and an equation of the plane is $-29(x+2) + 6(y-2) - 34(z-3) = 0$ or $-29x + 6y - 34z = -32$.