Group Quiz 5
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Math 153

Name: Key.

No work = no credit

1.) Does the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n-1}{n^2}$ converge absolutely, converge, or diverge? Justify your answer.

AST. $\lim_{n\to\infty} \frac{n-1}{n^2} = 0$ so it converges by the AST.

but

L.C.T. $\lim_{n\to\infty} \frac{\frac{n-1}{n^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2-n}{n^2} = 1$ so $\sum \frac{n-1}{n^2}$

shares the same fate as $\sum \frac{1}{n}$ which diverges.

Hence it is conditionally convergent.

2.) Does the series $\sum_{n=2}^{\infty}(-1)^{n}\left(\frac{\ln n}{\ln n^2}\right)^n$ converge absolutely, converge, or diverge? Justify your answer.

Root Test

$\lim_{n\to\infty} \sqrt[n]{\left|(-1)^n\left(\frac{\ln n}{2\ln n}\right)^n\right|} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}$ so it converges absolutely. by the root test

3.) Does the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{2^n n! n}$ converge absolutely, converge, or diverge? Justify your answer.

ratio test

$$\lim_{n\to\infty} \frac{(2(n+1))!}{2^{n+1}\cdot(n+1)!\cdot(n+1)} \cdot \frac{2^n n!\, n}{(2n)!}$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)\cancel{(2n)!}\,\cancel{2^n}\,\cancel{n!}\, n}{\cancel{2^n}\cdot 2\cdot(n+1)\cdot\cancel{n!}\,(n+1)\cancel{(2n)!}}$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)n}{2(n+1)^2}$$

$$= \infty$$

Diverges by the ratio test