

Group Quiz 4
Dusty Wilson
Math 153 – Spring 2012

Name: KEY

No work = no credit

- 1.) Decide whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$ converges or diverges. If it converges, find the sum. Justify your answer.

$$\frac{1}{n^2 + 5n + 6} = \frac{1}{n+2} - \frac{1}{n+3}$$

$$\text{Sum} = \sum_{n=1}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3} \right) = \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \dots = \frac{1}{3}$$

- 2.) Decide whether the series $\sum_{n=1}^{\infty} \frac{3(-1)^{-n}}{\pi^n}$ converges or diverges. If it converges, find the sum. Justify your answer.

$$\begin{aligned} \sum_{n=1}^{\infty} 3 \left(\frac{-1}{\pi} \right)^n &= \sum_{n=1}^{\infty} 3 \left(\frac{-1}{\pi} \right) \left(\frac{-1}{\pi} \right)^{n-1} \\ &= \frac{-3/\pi}{1 - (-1/\pi)} \\ &= \frac{-3}{\pi + 1} \end{aligned}$$

3.) How many terms of the convergent p -series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ must be summed to obtain an approximation that is within 0.001 of the exact value of the series?

$$\int_N^{\infty} \frac{1}{x^2} dx \leq 0.001$$

$$\Rightarrow \lim_{t \rightarrow \infty} \left[-\frac{1}{x} \right]_N^t \leq 0.001$$

$$\Rightarrow \lim_{t \rightarrow \infty} \left(-\frac{1}{t} + \frac{1}{N} \right) \leq 0.001$$

$$\Rightarrow \frac{1}{N} \leq 0.001$$

$$\Rightarrow 1000 \leq N.$$