Group Quiz 2
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Math 153

Name: Key.

No work = no credit

1.) Find the coordinates of the point that is a distance of 2 from $(5,0,0)$ along the curve $\vec{r}(t)=\langle 5\cos t, 3\sin t, 4\sin t\rangle$ in the direction of increasing t.

$t=0$

$$|\vec{r}\,'(t)| = |\langle -5\sin t, 3\cos t, 4\cos t\rangle|$$
$$= \sqrt{25\sin^2 t + 9\cos^2 t + 16\cos^2 t}$$
$$= 5$$

$\Rightarrow$ solve $\int_0^t 5\,dt = 2$

$\Rightarrow 5t = 2$

$\Rightarrow t = 2/5$

$\Rightarrow$ the point is $\left(5\cos\frac{2}{5}, 3\sin\frac{2}{5}, 4\cos\frac{2}{5}\right)$

2.) Given that the old derivative relationship between position, velocity, and acceleration carries over to vector valued functions, find the velocity and position vector functions if the acceleration is $\vec{a}(t)=\langle\cos(t), 2\sin(t)\rangle$ with initial velocity $\vec{v}_0=\langle 1,3\rangle$ and initial position $\vec{r}(0)=\langle 1,4\rangle$

$$\vec{v}(t) = \langle \sin t + a, -2\cos t + b\rangle$$
$$= \langle \sin t + 1, -2\cos t + 5\rangle$$

$$\vec{r}(t) = \langle -\cos t + t + c, -2\sin t + 5t + d\rangle$$
$$= \langle -\cos t + t + 2, -2\sin t + 5t + 4\rangle$$

and $\vec{r}(\pi) = \langle 3+\pi, 5\pi+4\rangle$

3.) Given $\bar{r}(t) = \langle e^{2t}, e^{2t}\cos(t), e^{2t}\sin(t)\rangle$, find the equation of the osculating plane and kissing circle of the curve at $(1, 1, 0)$.

$$\vec{r}'(t) = \langle 2e^{2t}, \underbrace{2e^{2t}\cos t - e^{2t}\sin t}_{e^{2t}(2\cos t - \sin t)}, \underbrace{2e^{2t}\sin t + e^{2t}\cos t}_{e^{2t}(2\sin t + \cos t)}\rangle\Big|_{t=0} \langle 2, 2, 1\rangle$$

$$\vec{r}''(t) = \langle 4e^{2t}, 2e^{2t}(2c - s) + e^{2t}(-2s - c), 2e^{2t}(2s + c) + e^{2t}(2c - s)\rangle$$

$$= \langle 4e^{2t}, e^{2t}(3c - 4s), e^{2t}(3s + 4c)\rangle\Big|_{t=0} \langle 4, 3, 4\rangle$$

$$|\vec{r}'(t)| = \sqrt{4e^{4t} + e^{4t}(4c^2 - 4cs + s^2) + e^{4t}(4s^2 + 4cs + c^2)}$$

$$= \sqrt{e^{4t}(4 + 5\cos^2 t + 5\sin^2 t)}$$

$$= 3e^{2t}\Big|_{t=0} 3$$

$$\Rightarrow \vec{T}(t) = \langle \tfrac{2}{3}, \tfrac{1}{3}(2c - s), \tfrac{1}{3}(2s + c)\rangle\Big|_{t=0} \langle \tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}\rangle$$

$$\vec{T}'(t) = \langle 0, \tfrac{1}{3}(-2s - c), \tfrac{1}{3}(2c - s)\rangle$$

$$|\vec{T}'(t)| = \sqrt{0 + \tfrac{1}{9}(4s^2 + 4cs + c^2) + \tfrac{1}{9}(4c^2 - 4cs + s^2)}$$

$$= \frac{\sqrt{5}}{3}$$

$$\Rightarrow \vec{N}(t) = \langle 0, \tfrac{1}{\sqrt{5}}(-2s - c), \tfrac{1}{\sqrt{5}}(2c - s)\rangle\Big|_{t=0} \langle 0, -\tfrac{1}{\sqrt{5}}, \tfrac{2}{\sqrt{5}}\rangle$$

$$\Rightarrow B(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2/3 & 2/3 & 1/3 \\ 0 & -1/\sqrt{5} & 2/\sqrt{5} \end{vmatrix} = \langle \frac{\sqrt{5}}{3}, \frac{-4}{3\sqrt{5}}, \frac{-2}{3\sqrt{5}}\rangle$$

To find curvature when $t = 0$

$$\vec{r}'(0) \times \vec{r}''(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{vmatrix} = \langle 5, -4, -2\rangle \text{ w/ norm } \sqrt{45}$$

so the curvature is

$$k = \frac{\sqrt{45}}{} = \frac{\sqrt{5}}{}$$

osculating plane @ $t=0$

$$\frac{\sqrt{5}}{3}(x-1) - \frac{4}{3\sqrt{5}}(y-1) - \frac{2}{3\sqrt{5}}(z-0) = 0$$

kissing circle

$$\vec{r}(t) = \frac{9}{\sqrt{5}}\cos(t)\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle + \frac{9}{\sqrt{5}}\sin(t)\left\langle 0, \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

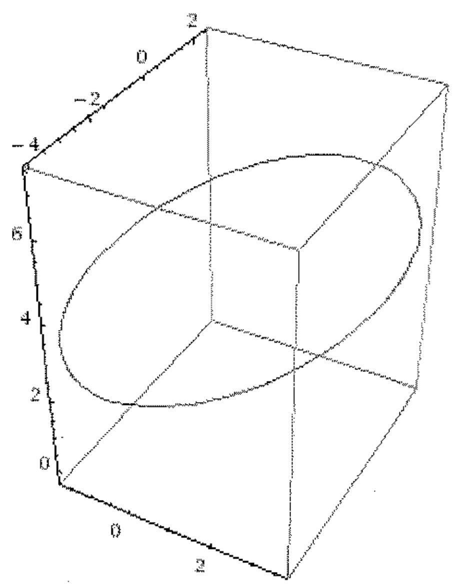
$$+ \langle 1, 1, 0 \rangle + \frac{9}{\sqrt{5}}\left\langle 0, \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

If you would like to check your graph, enter your kissing circle into WolframAlpha. Here is the result that I got. Use the command "ParametricPlot3D[*equation* , {t, 0, 2 Pi}]". You can type $\sqrt{5}$ as Sqrt[5] and type vectors using "{ }" instead of "< >".

Input interpretation:

3D parametric plot	$1+\frac{6\cos(t)}{\sqrt{5}}$ $-\frac{4}{5}+\frac{6\cos(t)}{\sqrt{5}}-\frac{9\sin(t)}{5}$ $\frac{18}{5}+\frac{3\cos(t)}{\sqrt{5}}+\frac{18\sin(t)}{5}$	$t = 0$ to 2π

Parametric plot:



Enable interactivity

Arc length of parametric curve:

More digits

$$\int_0^{2\pi} \frac{9}{\sqrt{5}}\,dt = \frac{18\pi}{\sqrt{5}} \approx 25.2893$$