Group Quiz 1
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Math 153

Name: KEY

No work = no credit

1.) Find the parametric equation of the line of intersection of the planes $3x-2y+z=1$ and $2x+y-3z=3$.

$\vec{n}_1 = \langle 3, -2, 1 \rangle$

$\vec{n}_2 = \langle 2, 1, -3 \rangle$

$\vec{n}_1 \times \vec{n}_2 = \langle 5, 11, 7 \rangle$ is parallel to both planes and thus in the direction of the line of intersection.

Find a pt of intersection. Set $z = 0$.

$3x - 2y = 1$
$2x + y = 3$
$\Rightarrow$ $x = 1$, $y = 1$ $\rightarrow$ pt $(1, 1, 0)$

line of intersection

$\vec{r}(t) = \langle 1, 1, 0 \rangle + t \langle 5, 11, 7 \rangle$

2.) Find the equation of the plane (A) that passes through the points $(0,-2,5)$ and $(-1,3,1)$ and is perpendicular to the plane (B) through the points $(0,0,0)$, $(2,1,7)$, and $(2,-2,1)$.

Find a normal to plane B.

$$\vec{n}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 7 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= \langle 15, 12, -6 \rangle$$

A third pt on A is $(15, 10, -1)$ and so a normal to plane A is

$$\vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 15 & 12 & -6 \\ -1 & +5 & -4 \end{vmatrix} = \langle -18, 66, +87 \rangle$$

An eqt of the plane A is

$$-18(x-0) + 66(y+2) + 87(z-5) = 0.$$

3.) Find the distance between the skew lines (skew lines are not parallel and do not intersect) with parametric equations $\vec{r}_1(t) = \langle 1+t, 1+6t, 2t \rangle$ and $\vec{r}_2(s) = \langle 1+2s, 5+15s, -2+6s \rangle$

The lines are on parallel planes. These are intersected by planes of normals $\vec{n}_1, \vec{n}_2$:

$$\vec{n}_1 = \langle 1, 6, 2 \rangle$$

$$\vec{n}_2 = \langle 2, 15, 6 \rangle$$

so $\vec{n}_1 \times \vec{n}_2 = \langle 6, -2, 3 \rangle \leftarrow$ parallel to the line of intersection & $\perp$ to the parallel planes.

Plane 1 has eqt: $6(x-1) + -2(y-1) + 3(z-0) = 0$

pt on plane 2: $(1, 5, -2)$ on the second line.

The dist from the pt to the plane: $\dfrac{|\langle 1,5,-2 \rangle \cdot \langle 6,-2,3 \rangle - 4|}{|\langle 6,-2,3 \rangle|}$

$$\Rightarrow D = \frac{14}{7} = 2$$