Quiz 5 key

$$\sum_{n=1}^{\infty} \frac{(1-2x)^n}{2n \cdot 3^n}$$

ratio test

$$\lim_{n\to\infty} \left| \frac{(1-2x)^{n+1}}{2(n+1)\, 3^{n+1}} \cdot \frac{2n\ 3^n}{(1-2x)^n} \right| < 1$$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{(1-2x)\cdot 2n}{(2n+2)\cdot 3} \right| < 1$$

$$\Rightarrow \left| \frac{1-2x}{3} \right| < 1$$

$$\Rightarrow |1-2x| < 3$$

$$\Rightarrow -3 < 1-2x < 3$$

$$\Rightarrow -4 < -2x < 2$$

$$\Rightarrow 2 > x > -1$$

check endpoints.

$x = 2$. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{2n\cdot 3^n} = \sum_{n=1} \frac{(-1)^n}{2n}$ convergent alt. series.

$x = -1$ harmonic series

I.O.C. $(-1, 2]$ & $R = \frac{3}{2}$.

$$\int_0^1 \frac{1-\cos x}{x^2}\,dx = \int_0^1 \frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots\right)}{x^2}\,dx$$

$$= \int_0^1 \left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} + \dots\right) dx$$

$$= \int_0^1 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!} x^{2(n-1)}\,dx$$

$$= \left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!} \frac{x^{2n-1}}{(2n-1)}\right]_0^1$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!} \frac{1}{(2n-1)}$$

find n s.t. $\frac{1}{(2(n+1))!\,(2(n+1)-1)} \le 0.00000001$

$\Rightarrow \boxed{n=5}$

$$\int_0^1 \frac{1-\cos x}{x^2}\,dx \approx \sum_{n=1}^{5} \frac{(-1)^{n+1}}{(2n)!\,(2n-1)} = 0.486353764$$

$f(x) = 2^x$ about $a=1$

f	2^x	2
f'	$\ln 2 \cdot 2^x$	$2\ln 2$
f''	$(\ln 2)^2 \cdot 2^x$	$2(\ln 2)^2$
$f^{(n)}$	$(\ln 2)^n \cdot 2^x$	$2(\ln 2)^n$

$$2^x = \frac{2(x-1)^0}{0!} + \frac{2\ln 2\,(x-1)^1}{1!} + \frac{2(\ln 2)^2(x-1)^2}{2!} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{2(x-1)^n(\ln 2)^n}{n!}$$

what is the minimum number of terms guaranteed so that the approx will be w/in 0.01 of $f(2)$.

For $0 \le x \le 2$, $|f^{(n+1)}(x)| \le (\ln 2)^{n+1} \cdot 2^2 = M$ on $0 \le x \le 2$

and $|R_n(x)| \le \dfrac{(\ln 2)^{n+1} 2^2 \cdot |x-1|^{n+1}}{(n+1)!} \le 0.01$ on $0 \le x \le 2$

$\Rightarrow \dfrac{(\ln 2)^{n+1} \cdot 4}{(n+1)!} \le 0.01$

$\Rightarrow n = 4$

$$2^2 \approx 3.99699 \approx \sum_{n=0}^{4} \frac{2(x-1)^n(\ln 2)^n}{(n)!}$$