Group Quiz 4
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Math 153 – Spring 2012

Name: KEY

No work = no credit

1.) Consider the three infinite series:

a.) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5n}$	b.) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$	c.) $\sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}}$

i.) Which of these series is (are) alternating?

(a.) & (c.) are alternating.

ii.) Which one of these series diverges, and why?

(b.) is divergent. Since the limit of the terms $\neq 0$ it diverges by the test for divergence.

iii.) One of these series converges absolutely. Which one? Compute the sum.

(c.) converges.

$$\sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}} = \sum_{n=1}^{\infty} \frac{5 \cdot 16 \cdot (-4)^n}{3 \cdot 9^n} = \frac{\frac{-320}{27}}{1-\left(\frac{-4}{9}\right)} = \frac{-320}{39}$$

Geometric
1st term: $\frac{-320}{27}$

ratio: $\frac{-4}{9}$

2.) Decide whether the series $\sum_{n=1}^{\infty} \frac{1}{2^n - n}$ converges or diverges. Justify your answer.

$$\lim_{n\to\infty} \frac{\frac{1}{2^n}}{(2^n - n)^{-1}} = \lim_{n\to\infty} \frac{2^n - n}{2^n}$$

$$= \lim_{n\to\infty} \left(1 - \frac{n}{2^n}\right)$$

$$= 1.$$

Since $\sum_{n=1}^{\infty} \frac{1}{2^n}$ is a convergent geometric series, the series converges by the limit comparison test.

3.) Decide whether the series $\sum_{n=1}^{\infty} \frac{n^n}{(n!)!}$ converges or diverges. Justify your answer.

$$\lim_{n\to\infty} \frac{(n+1)^{n+1}}{((n+1)!)!} \cdot \frac{(n!)!}{n^n}$$

Hence it converges by the ratio test.

$$= \lim_{n\to\infty} \frac{(n+1)(n+1)^n (n!)!}{n^n \cdot \cancel{[(n+1)! \cdot ((n+1)!-1) \cdots] \cdot [\cdots 2 \cdot 1]}}$$

$$((n+1)\, n!)!$$

$$\leq \lim_{n\to\infty} \frac{(n+1)(n+1)^n \cancel{(n!)!}}{n^n (n+1)! \cancel{(n!)!}}$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \cdot \frac{1}{n!}$$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \cdot \lim_{n\to\infty} \frac{1}{n!}$$

$$= e \cdot 0$$
$$= 0 < 1$$