Group Quiz 3
Dusty Wilson
Math 153 – Spring 2012

Name: KEY

No work = no credit

1.) Reparametrize the curve $\vec{r}(t)=\langle e^t,\ e^t\sin(t),\ e^t\cos(t)\rangle$ w/ the arclength measured from $(1,0,1)$.

$\vec{r}\,'(t) = \langle e^t,\ e^t\sin t + e^t\cos t,\ e^t\cos t - e^t\sin t\rangle$

$= e^t\langle 1,\ \sin t+\cos t,\ \cos t - \sin t\rangle$

$|\vec{r}\,'(t)| = \sqrt{(e^t)^2\left[1+\sin^2 t + \cancel{2\sin t\cos t} + \cos^2 t + \cos^2 t - \cancel{2\sin t\cos t} + \sin^2 t\right]}$

$= \sqrt{3}\,e^t$

$s(t) = \int_0^t \sqrt{3}\,e^u\,du$

$= \left[\sqrt{3}\,e^u\right]_0^t$

$= \sqrt{3}(e^t - 1)$

$\frac{s}{\sqrt{3}} = e^t - 1$

$\Rightarrow e^t = \frac{s}{\sqrt{3}} + 1$

$\Rightarrow t = \ln\left(\frac{s}{\sqrt{3}}+1\right)$

$\vec{r}(s) = \left\langle \frac{s}{\sqrt{3}}+1,\ \left(\frac{s}{\sqrt{3}}+1\right)\sin\left(\ln\left(\frac{s}{\sqrt{3}}+1\right)\right),\ \left(\frac{s}{\sqrt{3}}+1\right)\cos\left(\ln\left(\frac{s}{\sqrt{3}}+1\right)\right)\right\rangle$

2.) Find the tangential and normal components of the acceleration vector of a particle with position function $\vec{r}(t)=\langle t,\ 2t,\ t^2\rangle$

$\vec{r}\,'(t) = \langle 1, 2, 2t\rangle$

$\vec{r}\,''(t) = \langle 0, 0, 2\rangle$

$|\vec{r}\,'(t)| = \sqrt{5+4t^2}$

$\vec{r}\,'\cdot\vec{r}\,'' = 4t$

$\vec{r}\,'\times\vec{r}\,'' = \langle 4, -2, 0\rangle$

$|\vec{r}\,'\times\vec{r}\,''| = \sqrt{20}$

$a_T = \frac{4t}{\sqrt{5+4t^2}}$

$a_N = \frac{\sqrt{20}}{\sqrt{5+4t^2}}$

3.) Find the equation of the osculating circle of the curve $\vec{r}(t)=\langle \sin(2t),\ t,\ \cos(2t)\rangle$ at $(0,\ \pi,\ 1)$

$\vec{r}\,'(t) = \langle 2\cos 2t, 1, -2\sin 2t \rangle \Big|_{t=\pi} \quad \langle 2, 1, 0 \rangle$

$\vec{r}\,''(t) = \langle -4\sin 2t, 0, -4\cos 2t \rangle \Big|_{t=\pi} \quad \langle 0, 0, -4 \rangle$

$\vec{r}\,' \times \vec{r}\,'' = \langle -4, 8, 0 \rangle$ @ $t=\pi$

$|\vec{r}\,' \times \vec{r}\,''| = \sqrt{80}$ @ $t=\pi$

$|r'| = \sqrt{4\cos^2 2t + 1 + 4\sin^2 2t} = \sqrt{5}$ for all t.

$K = \dfrac{\sqrt{80}}{(\sqrt{5})^3} = \dfrac{4}{5}$ so the radius is $\boxed{R = \frac{5}{4}}$

$\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \left\langle \dfrac{2}{\sqrt{5}}\cos 2t, \dfrac{1}{\sqrt{5}}, -\dfrac{2}{\sqrt{5}}\sin 2t \right\rangle \Big|_{t=\pi}$ $\boxed{\begin{matrix}\vec{T}(\pi)\\ \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right\rangle\end{matrix}}$

$\vec{T}\,'(t) = \left\langle -\dfrac{4}{\sqrt{5}}\sin 2t, 0, -\dfrac{4}{\sqrt{5}}\cos 2t \right\rangle \Big|_{t=\pi} \quad \left\langle 0, 0, -\dfrac{4}{\sqrt{5}} \right\rangle$

$\boxed{\vec{N}(\pi) = \langle 0, 0, -1 \rangle}$

$\text{circle}(\theta) = R\cos\theta\,\vec{T} + R\sin\theta\,\vec{N} + \langle 0, \pi, 1 \rangle + R\vec{N}$

$= \left\langle \dfrac{\sqrt{5}}{2}\cos\theta, \dfrac{\sqrt{5}}{4}\cos\theta + \pi, -\dfrac{5}{4}\sin\theta + 1 + -\dfrac{5}{4} \right\rangle$