Group Quiz 1
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Math 153 – Spring 2012

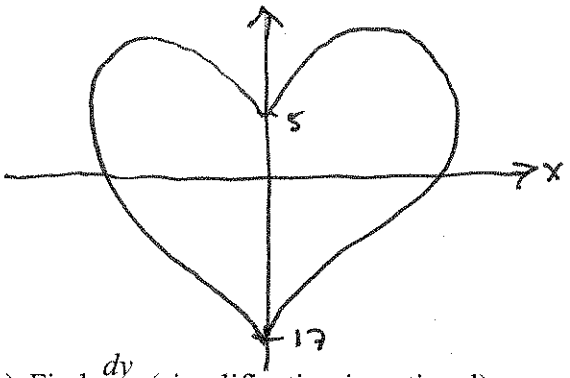
Name: KEY

No work = no credit

1.) Consider the parametrically curve:

$$x(t)=16\sin^3(t) \text{ and } y(t)=13\cos(t)-5\cos(2t)-2\cos(3t)-\cos(4t)$$

a.) Use your calculator and sketch a graph of the curve.



b.) Find $\frac{dy}{dx}$ (simplification is optional).

$$\frac{dy}{dx}=\frac{-13\sin t+10\sin 2t+6\sin 3t+4\sin 4t}{48\sin^2 t\cos t}$$

c.) Set up an equation to find where the tangents are horizontal. Clearly show where these exist on the graph in (a.).

$$-13\sin t+10\sin 2t+6\sin 3t+4\sin 4t=0$$

d.) Set up an equation to find where the tangent does not exist. Clearly show where these exist on the graph in (b.).

$$48\sin^2 t\cos t=0$$

e.) Set up (do not solve) an integral to represent the arclength of the figure.

$$L=\int_0^{2\pi}\sqrt{(48\sin^2 t\cos t)^2+(-13\sin t+10\sin 2t+6\sin 3t+4\sin 4t)^2}\,dt$$

2.) (No calculator) Carefully sketch a graph that includes $r = 1+\cos\theta$ and $r = 3\cos\theta$. Make sure to label each graph. Find the point(s) of intersection and express the coordinate(s) in polar form.

θ	① r	② r
0	2	3
$\pi/2$	1	0
π	0	-3
$\frac{3\pi}{2}$	1	0
2π	2	3

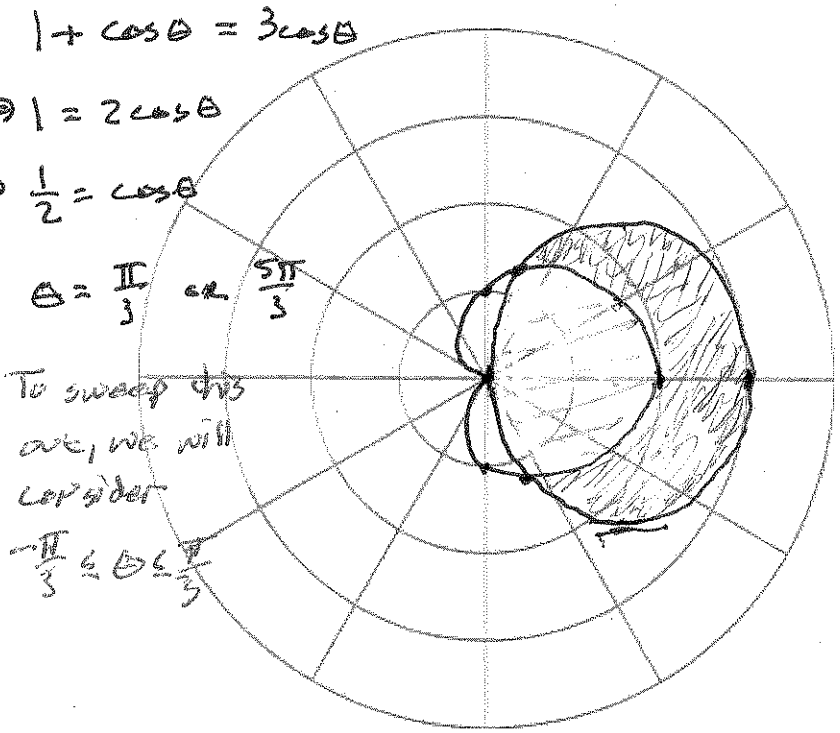
$1+\cos\theta = 3\cos\theta$

$\Rightarrow 1 = 2\cos\theta$

$\Rightarrow \frac{1}{2} = \cos\theta$

$\Rightarrow \theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

To sweep this out, we will consider $-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$



$r = 3\cos\theta$

$\Rightarrow r = 3\frac{x}{r}$

$\Rightarrow r^2 = 3x$

$\Rightarrow x^2 - 3x + \frac{9}{4} + y^2 = \frac{9}{4}$

3.) Find the area of the region bounded between the two curves in the previous question.

$$A = \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (3\cos\theta)^2 - (1+\cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 9\cos^2\theta - 1 - 2\cos\theta - \cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{8}{2}(1+\cos 2\theta) - 1 - 2\cos\theta \, d\theta$$

$$= \frac{1}{2}\left[3\theta + 2\sin 2\theta - 2\sin\theta \right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{2}\left[2\cdot\frac{\pi}{3} + 2\cdot\frac{\sqrt{3}}{2} - 2\cdot\frac{\sqrt{3}}{2} \right]$$

$$= \pi$$