13.4: Motion in Space: Velocity and Acceleration

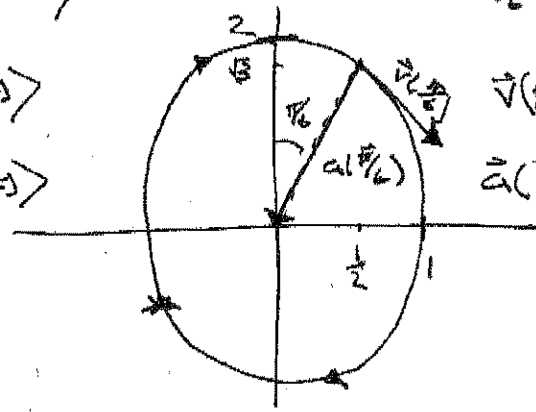
If $\vec{r}(t)$ gives the position of a particle moving thru space, then (as expected) $\vec{v}(t) = \vec{r}\,'(t)$ and $\vec{a}(t) = \vec{r}\,''(t)$.

<u>Ex1:</u> Find and sketch r, v, a at $t = \pi/6$ if

$\vec{r}(t) = \langle \sin(t), 2\cos(t) \rangle$.

$\vec{v}(t) = \langle \cos(t), -2\sin(t) \rangle$

$\vec{a}(t) = \langle -\sin(t), -2\cos(t) \rangle$



$\vec{r}(\frac{\pi}{6}) = \langle \frac{1}{2}, \sqrt{3} \rangle$

$\vec{v}(\frac{\pi}{6}) = \langle \frac{\sqrt{3}}{2}, -1 \rangle$

$\vec{a}(\frac{\pi}{6}) = \langle -\frac{1}{2}, -\sqrt{3} \rangle$

<u>Ex2:</u> Find $\vec{v}(t)$ and $\vec{r}(t)$ if $\vec{a}(t) = \langle 0, 0, 1 \rangle$ and $\vec{v}(0) = \langle 1, -1, 0 \rangle$ and $r(0) = \vec{0}$.

<u>Tangential and Normal components of Acceleration</u>

$\vec{v}$ gives velocity and we use $v = |\vec{v}|$ to represent speed.

Now, recall $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{\vec{v}(t)}{|\vec{v}(t)|} = \dfrac{\vec{v}}{v}$

$\Rightarrow \vec{v} = v\vec{T} \Rightarrow \vec{v}\,' = \vec{a} = v'\vec{T} + v\vec{T}\,'$ $*$

also recall $k = \dfrac{|\vec{T}\,'|}{|\vec{r}\,'|} = \dfrac{|\vec{T}\,'|}{v} \Rightarrow |\vec{T}\,'| = kv$.

but we defined the unit normal as $\vec{N} = \dfrac{\vec{T}\,'}{|\vec{T}\,'|}$

$\Rightarrow \vec{T}\,' = \vec{N}|\vec{T}\,'| = kv\vec{N}$

$*$ Hence $\vec{a} = v'\vec{T} + kv^2\vec{N}$

Now we can write

$$\vec{a} = a_T\vec{T} + a_N\vec{N} \quad \text{where } a_T = v' \text{ and } a_N = kv^2$$

Find a_T and a_N in terms of $\vec{r}$.

a_T: $\vec{v}\cdot\vec{a} = vT\cdot(v'\vec{T} + kv^2\vec{N})$

$= vv'\vec{T}\cdot\vec{T} + kv^3\,\vec{T}\cdot\vec{N}$ (since $\vec{T}\cdot\vec{T} = 1$, $\vec{T}\cdot\vec{N} = 0$)

$= v\,v'$

$$\Rightarrow a_T = v' = \frac{\vec{v}\cdot\vec{a}}{v} = \frac{\vec{r}'(t)\cdot\vec{r}''(t)}{|\vec{r}'(t)|}$$

a_N: $$kv^2 = \frac{|\vec{r}'(t)\times\vec{r}''(t)|}{|\vec{r}'(t)|^3}\,|\vec{r}'(t)|^2 = \frac{|\vec{r}'(t)\times\vec{r}''(t)|}{|\vec{r}'(t)|}$$

<u>Ex 2:</u> A particle moves along the twisted cubic $\vec{r}(t) = \langle t, t^2, t^3\rangle$. Find the tangential and normal components of acceleration.