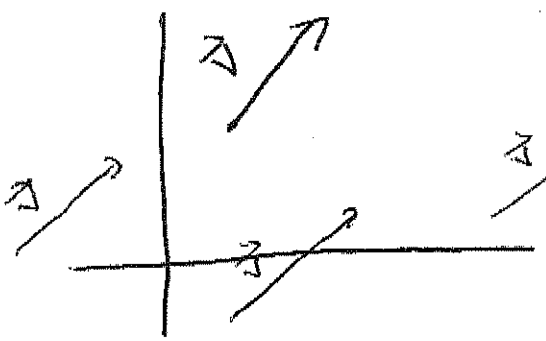
12.2: Vectors

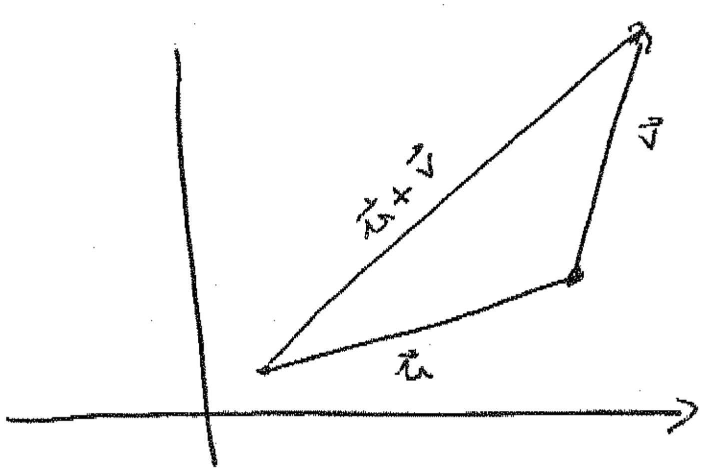
A vector is a quantity w/ both magnitude and direction.

graphically, they are represented by arrows.

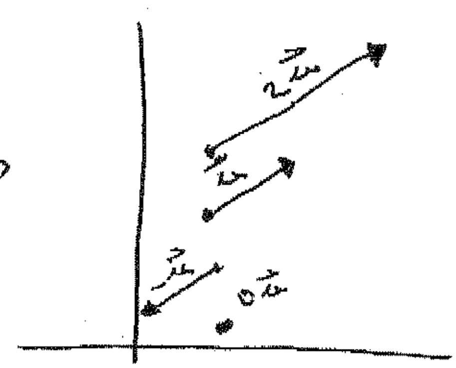


Just as the statement "travel 3mph NE" can be made irrespective of the point of origin, vectors are independent of their initial point

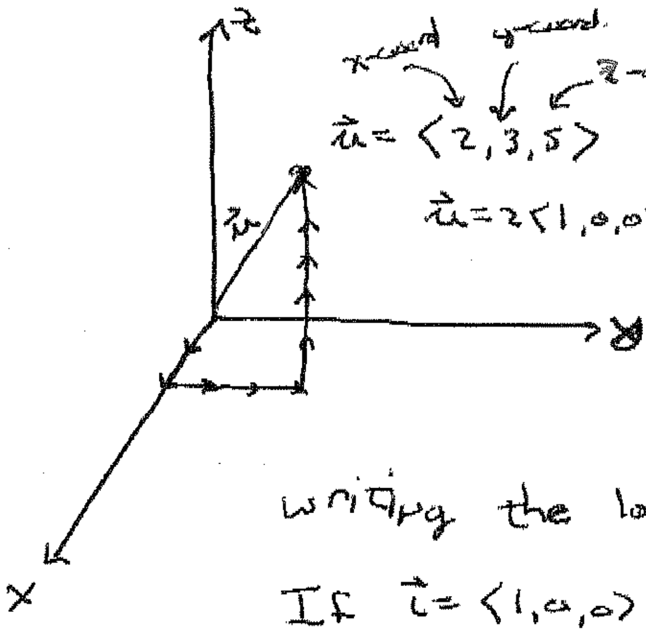
To add vectors graphically $(\vec{u}+\vec{v})$, we find the vector joining the initial point of $\vec{u}$ to the terminal point of $\vec{v}$ when the terminal point of $\vec{u}$ is the same as the initial point on $\vec{v}$.



similarly, we can scale the length of $\vec{u}$.



Algebraically, we list vectors by their components.



x-coord y-coord z-coord

$\vec{u} = \langle 2, 3, 5 \rangle$ OR

$\vec{u} = 2\langle 1,0,0\rangle + 3\langle 0,1,0\rangle + 5\langle 0,0,1\rangle$

writing the last statement is tedious.

If $\vec{i} = \langle 1,0,0\rangle$
$\vec{j} = \langle 0,1,0\rangle$
$\vec{k} = \langle 0,0,1\rangle$

then $\vec{u} = \langle 2, 3, 5\rangle = 2\vec{i} + 3\vec{j} + 5\vec{k}$

we call $\vec{i}, \vec{j}$, and $\vec{k}$ the standard basis vectors.

we could also say $\langle 2,3,5\rangle$ is the <u>position vector</u> for $P(2,3,5)$.

The vector $\vec{AB}$ when $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle$

Algebraically, find $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$.

Note: The vector $\overrightarrow{AB} = \vec{B} - \vec{A}$ (position vectors).

Note: vectors in $\mathbb{R}^n$ $\vec{u} = \langle u_1, u_2, \dots, u_n \rangle$

Properties of Vectors: If $\vec{u}, \vec{v}$, and $\vec{w} \in \vec{V}_n$ and a, b are scalars ...

1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2) $\vec{u} + \vec{0} = \vec{u}$
3) $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$
4) $(ab)\vec{u} = a(b\vec{u})$
5) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
6) $\vec{u} + (-\vec{u}) = \vec{0}$
7) $(a+b)\vec{u} = a\vec{u} + b\vec{u}$
8) $1 \cdot \vec{u} = \vec{u}$.

prove (1) and (3).

Norm or magnitude

Defn: $|\vec{u}| = \sqrt{u_1^2 + u_2^2 + \dots + u_n^2}$

on in $\vec{V}_3$, $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$.

Ex 1: Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u}$, and $3\vec{u} + 4\vec{v}$, and $|\vec{u}|$ when $\vec{u} = \langle -3, -4, -1 \rangle$ and $\vec{v} = \langle 6, 2, -3 \rangle$

Ex 2: Find a vector in the same direction as $\langle -2, 4, 2 \rangle$ w/ length 1 (unit vector) and w/ length 6.