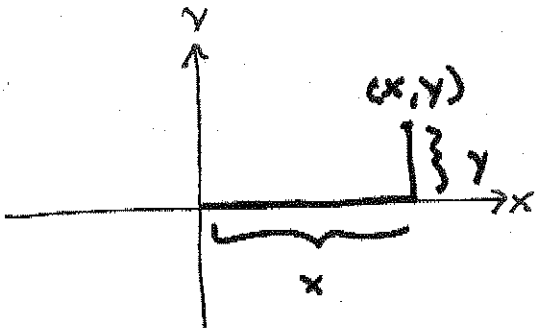
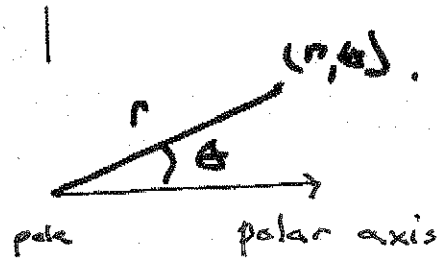
10.3 : Polar Coordinates

cartesian coords



polar coords



Ex1: Plot the polar point $(2, \frac{3\pi}{4})$ & find two other coords of this point. ($r>0$ & $r<0$).

Ex2: Find the cartesian coords of $(r,\theta) = (-2, -5\pi/6)$.

Ex3: sketch the region $2 < r \le 5$ and $\frac{3\pi}{4} < \theta < \frac{5\pi}{4}$

To convert $x = r\cos\theta$, $y = r\sin\theta$, $\tan\theta = \frac{y}{x}$
and $r^2 = x^2 + y^2$.

Ex4: convert $r = 2\sin\theta + 4\cos\theta$ to a cart. eqt.

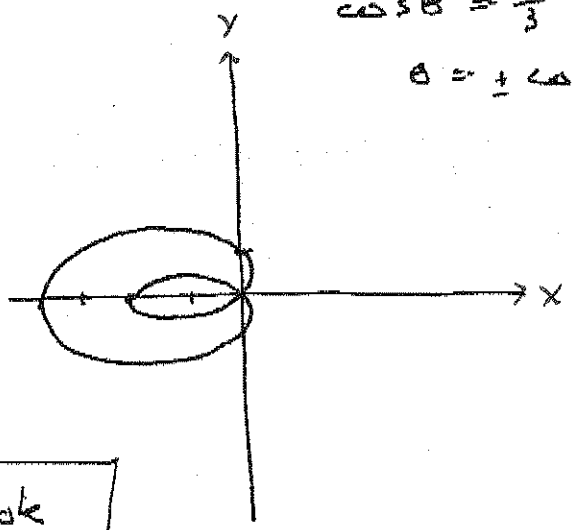
$$\Rightarrow r^2 = 2x + 4y$$
$$\Rightarrow x^2 - 2x + y^2 - 4y = 0$$
$$\Rightarrow (x-1)^2 + (y-2)^2 = 5$$

Ex 5: Sketch $r = 1 - 3\cos\theta$. $\Rightarrow$ $0 = 1 - 3\cos\theta$

$\cos\theta = \frac{1}{3}$

$\theta = \pm \cos^{-1}\left(\frac{1}{3}\right)$

θ	r
0	-2
$\frac{\pi}{2}$	1
π	4
$\frac{3\pi}{2}$	1



mathematica Notebook

Generally, we write $r = f(\theta)$.

$\Rightarrow$ $x = f(\theta)\cos\theta$, $y = f(\theta)\sin\theta$.

$\Rightarrow$ $\frac{dx}{d\theta} = f'(\theta)\cos(\theta) - f(\theta)\sin(\theta) = r'c - rs$

and $\frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta = r's + rc$

Now $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Ex 6: Find where the fat $r = 1 - 3\cos\theta$ has horizontal & vertical tangents

Ex 7: Find the tangent line when $\theta = \pi/3$.