50's	60's	70's	80's	90's	100+
2	1	4	6	3	2

Test 2
Dusty Wilson
Math 220

$\bar{x} = 82.1\%$
med = 85.3%

Name: key

Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties.

Evariste Galois
1811 – 1832 (French mathematician)

No work = no credit

Warm-ups (1 pt each)[1]: $\vec{e}_2 + \vec{e}_1 = \begin{bmatrix}1\\1\end{bmatrix}$ $\vec{e}_1\vec{e}_1^T = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ $\vec{e}_2^T\vec{e}_2 = [1]$

$\begin{bmatrix}1\\0\end{bmatrix}[1\ 0]$

1.) (1 pt) According to Galois, how do authors most hurt their readers? Answer using complete English sentences.

Authors aren't forthright w/their shortcomings.

2.) (10 pts) Find all eigenvalues of $A = \begin{bmatrix}-1 & 0 & 2\\3 & 0 & -6\\1 & 0 & -2\end{bmatrix}$ and state their algebraic multiplicity.

$$\begin{vmatrix}-1-\lambda & 0 & 2\\3 & -\lambda & -6\\1 & 0 & -2-\lambda\end{vmatrix} = -\lambda\begin{vmatrix}-1-\lambda & 2\\1 & -2-\lambda\end{vmatrix}$$

$$= -\lambda\left((-1-\lambda)(-2-\lambda) - 2\right)$$

$$= -\lambda\left(\cancel{2} + \lambda + 2\lambda + \lambda^2 - \cancel{2}\right)$$

$$= -\lambda(\lambda^2 + 3\lambda)$$

$$= -\lambda^2(\lambda+3)$$

$\lambda = 0$ (alg. mult 2)
$\lambda = -3$ (alg. mult 1)

1 pt.

median

	2015	2016
T1	70.7%	78.6%
T2	72.5%	85.3%

[1] In the warm-ups, $\vec{e}_i$ refers to the standard basis (column) vector in $\mathbb{R}^2$.

3.) (10 pts) Find the determinant of $B = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 3 & -1 \\ 3 & -1 & -1 & 2 \end{bmatrix}$

$$|B| = 1 \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & -1 \\ -1 & -1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 2 & -1 & 1 \\ -1 & 3 & -1 \\ 3 & -1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 3 \\ 3 & -1 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix}$$

$$-1 \left(2 \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix} \right)$$

$$-3 \left(2 \begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} \right)$$

$$= 5 + 3 + 1 - (10 + 1 - 8) - 3(2 + 8 + 5)$$

$$= 9 - 3 - 45$$

$$= -39$$

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Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties.

Evariste Galois
1811 – 1832 (French mathematician)

1.) (10 pts) The matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & -6 \\ 1 & 0 & -2 \end{bmatrix}$ has eigenvalues $\lambda = 0$ (with algebraic multiplicity 2) and $\lambda = -3$ (with algebraic multiplicity 1).

Find the corresponding (a.) eigenspaces, (b.) the geometric multiplicity of each eigenvalue, and if there is an eigenbasis (c.) diagonalize A.

(a.) $E_0 = \text{span}\left(\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$

$E_{-3} = \ker\left(\begin{bmatrix} 2 & 0 & 2 \\ 3 & 3 & -6 \\ 1 & 0 & 1 \end{bmatrix} \right) = \text{span}\left(\begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} \right)$

(b) the geometric multiplicity of $\lambda = 0$ is 2 while that of -3 is 1.

(c.) A is diagonalized by $B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -1 \end{bmatrix}$

w/ $S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

2.) (10 pts) Consider the matrix $A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$

a.) Find a basis for the image of A.

$$\operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for $\operatorname{im}(A)$: $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

b.) Find the kernel of A.

$x_1 = -x_3 + x_5$
$x_2 = -2x_3 - 3x_5$
$x_3 =$ free s
$x_4 = -4x_5$
$x_5 =$ free t

$$\Rightarrow \vec{x} = s\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix} \Rightarrow \ker(A) = \operatorname{span}\left(\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right)$$

c.) rank(A) = 3 and nullity(A) = 2

3.) (6 pts) Define the following:

a.) What does it mean if $\vec{v}_1, \dots, \vec{v}_m \in \mathbb{R}^n$ are linearly independent?

$c_1\vec{v}_1 + \dots + c_m\vec{v}_m = \vec{0}$ has only the trivial soln.

b.) What is the span of $\vec{v}_1, \dots, \vec{v}_m \in \mathbb{R}^n$.

The set of all linear combinations of $\vec{v}_1, \dots, \vec{v}_m$.

c.) How do we know if $\vec{v}_1, \dots, \vec{v}_m \in \mathbb{R}^n$ are a basis for a subspace V of $\mathbb{R}^n$?

They are a basis if they are L.I. and span V.

4.) (9 pts) Answer the following. 1 point per problem is for an explanation/justification:

a.) True or False, the image of a 3x4 matrix is a subspace of $\mathbb{R}^4$

False, the image is a subspace of $\mathbb{R}^3$,

b.) True or False, if $2\vec{u}+3\vec{v}+4\vec{w}=5\vec{u}+6\vec{v}+7\vec{w}$, then $\vec{u}$, $\vec{v}$, and $\vec{w}$ must be linearly dependent.

True: $\vec{0} = 3\vec{u} + 3\vec{v} + 4\vec{w}$ so there is a non-trivial sol^n to the homogenous eqt.

c.) True or False, if A and B are invertible 2x2 matrices, then AB must be similar to BA.

True : $\underset{S}{A^{-1}} AB = BA \underset{S}{A^{-1}}$

5.) (6 pts) According to our text, a subset W of the vector space $\mathbb{R}^n$ is called a <u>subspace</u> of $\mathbb{R}^n$ if it has the following three properties:

a.) $\vec{0} \in W$.

b.) $\vec{u}, \vec{v} \in W \Rightarrow \vec{u}+\vec{v} \in W$

closed under addition

c.) $\vec{u} \in W$ and $k \in \mathbb{R} \Rightarrow k\vec{u} \in W$.

closed under scalar multiplication.

d.) (2 pts extra credit) One of these conditions is not necessary, which one and why?

(a.) isn't necessary because it is addressed in (c.) if $k=0$.

6.) (10 pts) For the matrix A_{nxn}, there are at least 9 statements equivalent to, "*A* is invertible." List at least five of them. List more for 1 extra credit point (each).

i.) *A* is invertible.	vi.) rank$(A) = n$
ii.) $\det(A) \neq 0$	vii.) (1 pt extra credit) nullity$(A) = 0$
iii.) cols of A are L.I.	viii.) (1 pt extra credit) rref$(A) = I$
iv.) $\text{Im}(A) = \mathbb{R}^n$	ix.) (1 pt extra credit) 0 is not an eigen value.
v.) $\ker(A) = \vec{0}$	x.) (1 pt extra credit) cols of A span $\mathbb{R}^n$

cols of A are a basis for $\mathbb{R}^n$. $\quad A\vec{x} = \vec{b}$ has a unique sol'n $\forall\, \vec{b} \in \mathbb{R}^n$.

7.) (10 pts) Consider the linear transformation $T(\vec{x}) = A\vec{x}$ such that $T(\vec{v}_1) = 2\vec{v}_1$ and $T(\vec{v}_2) = -\vec{v}_2$ where $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

eigen vecs.

a.) Find the matrix *B* of the linear transformation

$$B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

b.) If $\vec{x} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$, find $[\vec{x}]_B$

$$[\vec{x}]_B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

c.) For the given $\vec{x}$, find $[T(\vec{x})]_B$

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \end{bmatrix} = [T(\vec{x})]_B$$

d.) For the given $\vec{x}$, find $T(\vec{x})$

$$T(\vec{x}) = -4\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$