Test 2
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Math 220

Name: Key

Nothing is more important than to see the sources of invention which are, in my opinion more interesting than the inventions themselves.

Gottfried Leibniz
1646 – 1716 (German mathematician)

No work = no credit

$\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}11\end{bmatrix}$ $\quad \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ $\quad A^{13}(A^{-1})^{6}=A^{7}$

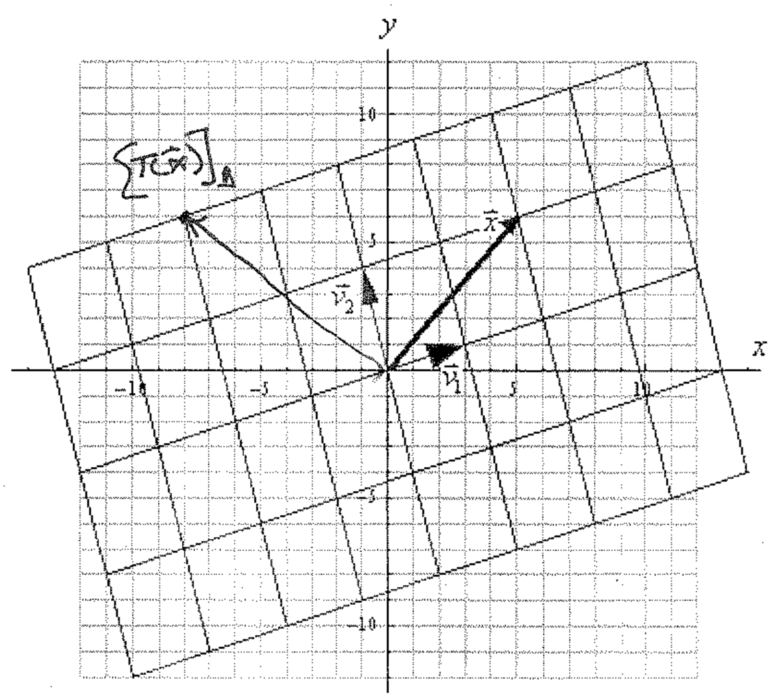
Warm-ups (1 pt each): $\cancel{A\theta}=$_____ $\quad \cancel{\theta^T\theta}=$_____ $\quad \cancel{\theta\theta^T}=$_____

1.) (1 pt) What did Leibniz find most interesting about inventions? (See above). Answer using complete English sentences.

The sources are most interesting.

2.) (5 pts) Use the given graph to answer the following:

a.) $\vec{v}_1=\begin{bmatrix}3\\1\end{bmatrix}$ and $\vec{v}_2=\begin{bmatrix}-1\\4\end{bmatrix}$



b.) $\vec{x}=\begin{bmatrix}5\\6\end{bmatrix}$

c.) $[\vec{x}]_B=\begin{bmatrix}2\\1\end{bmatrix}$

d.) If $T(\vec{x})=A\vec{x}$ and $A=\begin{bmatrix}-10/13 & -9/13\\-12/13 & 23/13\end{bmatrix}$, what is $[T(\vec{x})]_B=\begin{bmatrix}-2\\2\end{bmatrix}_B$

$T(\vec{x})=\begin{bmatrix}-8\\6\end{bmatrix}$

3.) (35 pts) Consider the matrix $A = \begin{bmatrix} 1 & -4 & 2 & 3 & -1 \\ 3 & -12 & 5 & 8 & -2 \\ 1 & -4 & 1 & 2 & 0 \end{bmatrix}$

a.) Find the image of A.

$$\text{rref}(A) = \begin{bmatrix} 1 & -4 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{im}(A) = \text{span}\left(\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right)$$

b.) Find the kernel of A.

$x_1 = 4x_2 - x_4 - x_5$

$x_2 = r$

$x_3 = -x_4 + x_5$

$x_4 = s$

$x_5 = t$

$$\vec{X} = r \underbrace{\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\vec{v}_1} + s \underbrace{\begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}_2} + t \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{v}_3}.$$

$\ker(A) = \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

c.) rank(A) = 2 and nullity(A) = 3

4.) (10 pts) Describe a basis, the conditions a basis must satisfy, and the meaning of those conditions.

A basis for a subspace V of $\mathbb{R}^n$ is a L.I, spanning, set of vectors in $\mathbb{R}^n$.

Vectors are L.I. if none are redundant ... that is a lin. comb. of the others.

vectors span a space if every vector in the space is a lin. comb. of the spanning vecs.

5.) (5 pts) Explain how you would go about proving an "if and only if" claim (sometimes denoted by "iff") that:

<u>Condition A is true</u> *if and only if* <u>Condition B is true</u>.

proof

($\Rightarrow$) Assume A

show that B follows.

($\Leftarrow$) Assume B

show that A follows.

$\therefore$ A iff B ∎

6.) (8 pts) Consider a linear transformation $T:\mathbb{R}^n \to \mathbb{R}^p$ and linearly dependent vectors $\vec{v}_1,\dots,\vec{v}_m \in \mathbb{R}^n$. Prove that $T(\vec{v}_1),\dots,T(\vec{v}_m)$ are linearly dependent as well.

$\square$ proof.

NTS: $c_1 T(\vec{v}_1) + \dots + c_m T(\vec{v}_m) = \vec{0}$ has non-trivial solutions.

We know $c_1 \vec{v}_1 + \dots + c_m \vec{v}_m = \vec{0}$ has non-triv. sol. (not all $c_i = 0$)

$\Rightarrow$ $T(c_1 \vec{v}_1 + \dots + c_m \vec{v}_m) = T(\vec{0})$ has sol. for at least one $c_i \neq 0$.

$\Rightarrow$ $c_1 T(\vec{v}_1) + \dots + c_m T(\vec{v}_m) = \vec{0}$ has non-triv. sol. $\blacksquare$

QED

(7) $\vec{x} = \begin{bmatrix} 3 \\ 7 \\ 13 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

and $[\vec{y}]_B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$\frac{5}{10}$ if S & S^{-1} swapped.

Find $[\vec{x}]_B = \begin{bmatrix} -1 \\ 4 \\ 10 \end{bmatrix}_B$

$[x]_B = S^{-1}\vec{x}$

$\frac{7}{10}$ is one right & one not

Find $\vec{y} = S[y]_B = \begin{bmatrix} 7 \\ 11 \\ 12 \end{bmatrix}$