2.3: Matrix Multiplication

recall: $\underset{n\times m}{A}\;\underset{m\times p}{B} = \begin{bmatrix} - & \vec{a}_1 & - \\ & \vdots & \\ - & \vec{a}_n & - \end{bmatrix}_{n\times m} \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_p \\ | & & | \end{bmatrix}_{m\times p}$

$$= \begin{bmatrix} \vec{a}_1\cdot\vec{b}_1 & \cdots & \vec{a}_1\cdot\vec{b}_p \\ \vdots & & \vdots \\ \vec{a}_n\cdot\vec{b}_1 & \cdots & \vec{a}_n\cdot\vec{b}_p \end{bmatrix}_{n\times p}$$

$$= \begin{bmatrix} | & & | \\ A\vec{b}_1 & \cdots & A\vec{b}_p \\ | & & | \end{bmatrix}_{n\times p}$$

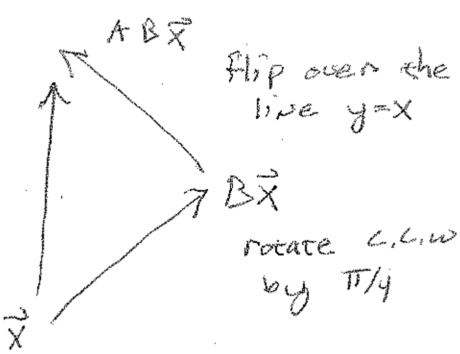
This form is what allows the bug. example to work

NOTE: The text introduces this by continuing their delightful coast guard example from 2.1.

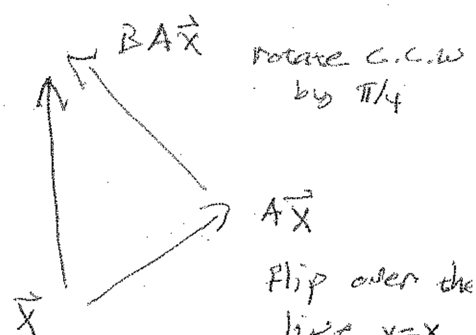
matrix multiplication is not commutative.

ex 1: If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) \end{bmatrix}$

compute & compare AB and BA. Interpret the answers geometrically using composition diagrams.



vs.



(see bug pics).

```
In[46]:= (A := {{0, 1}, {1, 0}}) // MatrixForm;
         (B := {{Cos[π/4], -Sin[π/4]}, {1, Sin[π/4]}}) // MatrixForm;

         GraphicsGrid[{{Show[Graphics[Line[Transpose[Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "Bug"],
             Show[Graphics[Line[Transpose[Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "Bug"]},
           {Show[Graphics[Line[Transpose[B.Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "B"."Bug"],
             Show[Graphics[Line[Transpose[A.Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "A"."Bug"]},
           {Show[Graphics[Line[Transpose[A.B.Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "A"."B"."Bug"],
             Show[Graphics[Line[Transpose[B.A.Transpose[Bug]]]],
              PlotRange → {{-50, 90}, {-50, 50}}, PlotLabel → "B"."A"."Bug"]}}]
```

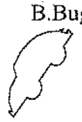
Bug

Bug

rotate c.c.w by $\pi/4$

flip over the line $y=x$

B.Bug



A.Bug

flip over the line $y=x$.

rotate c.c.w. by $\pi/4$

A.B.Bug

B.A.Bug

Some properties of matrix multiplication/algebra

Thm: multiplying w/ the identity matrix

For $A_{n \times m}$: $A I_m = I_n A = A$

Thm: matrix mult. is associative

$(AB)C = A(BC)$

Thm: matrix mult. is distributive

(a) $A(C+D) = AC + AD$

(b) $(A+B)C = AC + BC$.

$\square$ proof of (a)

Let $A_{n \times m}$ and $C, D_{m \times p}$.

$$C + D = \begin{bmatrix} | & & | \\ \overrightarrow{(c_1 + d_1)} & \cdots & \overrightarrow{(c_p + d_p)} \\ | & & | \end{bmatrix}$$

so
$$A(C+D) = \begin{bmatrix} | & & | \\ A(\vec{c}_1 + \vec{d}_1) & \cdots & A(\vec{c}_p + \vec{d}_p) \\ | & & | \end{bmatrix}$$

$$= \begin{bmatrix} | & & | \\ (A\vec{c}_1 + A\vec{d}_1) & \cdots & (A\vec{c}_p + A\vec{d}_p) \\ | & & | \end{bmatrix}$$

$$= \begin{bmatrix} | & & | \\ A\vec{c}_1 & \cdots & A\vec{c}_p \\ | & & | \end{bmatrix} + \begin{bmatrix} | & & | \\ A\vec{d}_1 & \cdots & A\vec{d}_p \\ | & & | \end{bmatrix}$$

$$= AC + AD \quad \blacksquare$$

<u>Thm:</u> If $A_{n\times p}$ and $B_{p\times m}$ and k a scalar

$$(kA)B = A(kB) = k(AB).$$

we are going to skip over block-matrices.