Gauss - Jordan Elimination.

ex1: $\begin{cases} x + 2y = 3 \\ 3x + 4y = 5 \end{cases}$

ex2: $\begin{cases} -2x + 3y = -13 \\ x + 5y = -13 \end{cases}$

Definition: Algorithm (Berlinski, 2000)

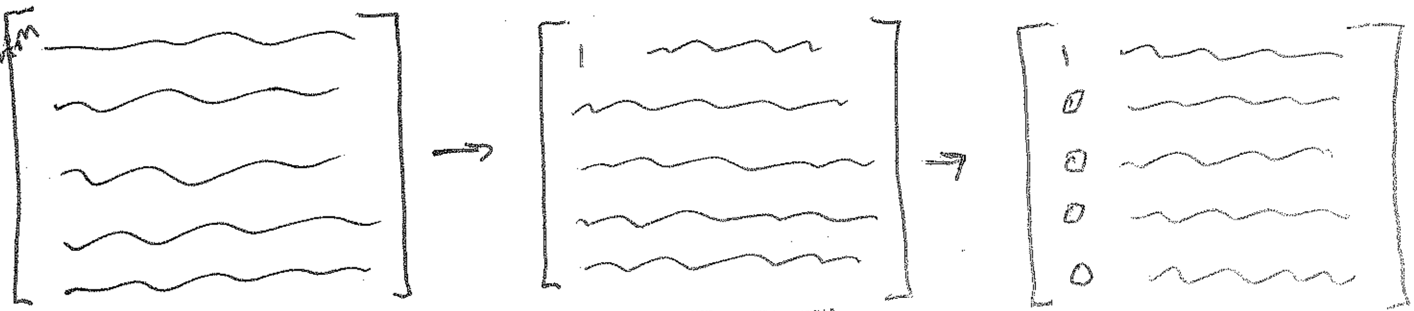
An algorithm is a finite procedure, written in a fixed symbolic vocabulary, governed by precise instructions, moving in discrete steps 1, 2, 3...whose execution requires no insight, cleverness, intuition, intelligence, or perspicuity, and that sooner or later comes to an end.

Note: Something is perspicuous if it is plain to the understanding especially because of clarity and precision of presentation.

GAUSS-JORDAN ELIMINATION OVERVIEW

TOOLS: Divide, subtract, and swap.

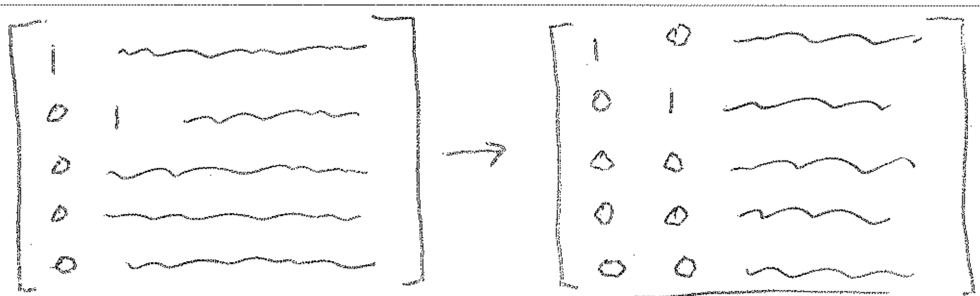
THE ALGORITHM



start

leading 1 or pivot

col of zeros w/pivot

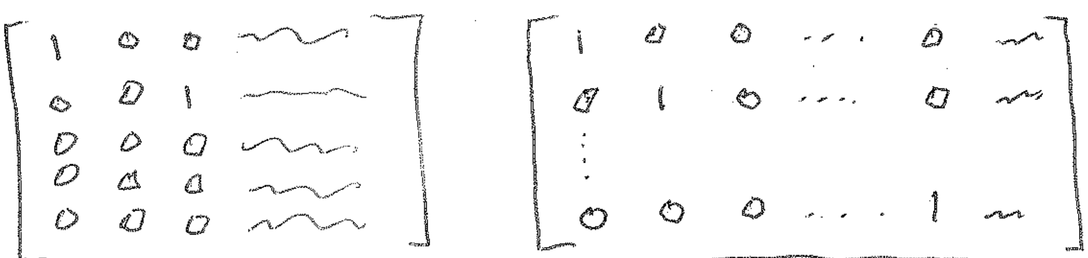


pivot

col of zeros w/pivot

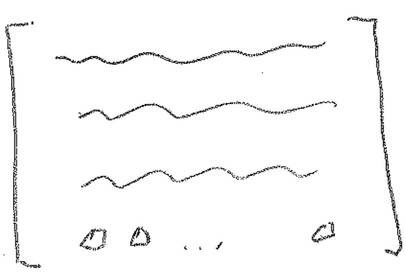
etc.

HOW IT ENDS



col. of zeros are possible

pivot in last row is ok



rows of zeros at bottom ok

examples of Gauss-Jordan Elimination

Tools: Row swap, ADD, scalar mult.

$$\left|\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y \quad\quad &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}\right| \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

w/ unique sol. $(1, -1, 2)$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 2 & -3 \\ 1 & 2 & -1 & 0 & 2 \\ 2 & 4 & 1 & -3 & -2 \\ 2 & 5 & 2 & -5 & -5 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

solution $(2 - t, -1 + t, -2 + t, t)$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 2 & 1 \\ 2 & -2 & -1 & 3 & 3 \\ -1 & 1 & -1 & 0 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

solution $(2 + s - t, s, 1 + t, t)$

or
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

(see supplemental algorithms)

1.2: Matrices, Vectors, and Gauss-Jordan Elimination (pre-reading assignment)

Vectors and vector spaces

A matrix with only one column is called a column vector, or simply a vector. The entries of a vector are called its components. The set of all column vectors with n components is denoted by $\mathbb{R}^n$; we will refer to $\mathbb{R}^n$ as a vector space. A matrix w/only one row is called a row vector. In this text, the term vector refers to column vectors, unless otherwise stated. The reason for our preference for column vectors will become apparent in the next section.

Standard representation of vectors

The standard representation of a vector $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ in the Cartesian coordinate plane is as an arrow (a directed line segment from the origin to the point (x, y). The standard representation of a vector in $\mathbb{R}^3$ is defined analogously. In this text, we will consider the standard representation, unless stated otherwise.

Reduced Row - Echelon Form

A matrix is in reduced row-echelon form if it satisfies all of the following conditions:

a.) If a row has nonzero entries, then the first nonzero entry is a 1, called the leading 1 (or pivot) in this row.

b.) If a column contains a leading 1, then all the other entries in that column are 0.

---

c.) If a row contains a leading 1, then each row above it contains a leading 1 further to the left.

condition c implies that rows of 0's, if any, appear @ the bottom of the matrix.

Types of elementary row operations

- Divide a row by a nonzero scalar.
- subtract a multiple of a row from another row
- Swap two rows.