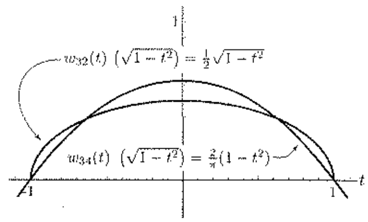
b. For $f(t) = \sqrt{1-t^2}$ we have $\|f\|_{32} = \sqrt{\frac{1}{2}\int_{-1}^{1}(1-t^2)\,dt} = \sqrt{2/3}$ and $\|f\|_{34} = \sqrt{\langle \sqrt{1-t^2}, \sqrt{1-t^2}\rangle_{34}} = \sqrt{\langle 1, 1-t^2\rangle_{34}} = \sqrt{1-1/4} = \sqrt{3/4}$.




## True or False

**Ch 5.TF.1** F. Consider $T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\vec{x}$.

**Ch 5.TF.2** T, by Theorem 5.3.9.b

**Ch 5.TF.3** T, by Theorem 5.3.4a

**Ch 5.TF.4** F. We have $(AB)^T = B^T A^T$, by Theorem 5.3.9a.

**Ch 5.TF.5** T, since $(A+B)^T = A^T + B^T = A + B$

**Ch 5.TF.6** T, by Theorem 5.3.4

**Ch 5.TF.7** F. Consider $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

**Ch 5.TF.8** T. First note that $A^T = A^{-1}$, by Theorem 2.4.8. Thus $A$ is orthogonal, by Theorem 5.3.7.

**Ch 5.TF.9** F. The correct formula is $\text{proj}_L(\vec{x}) = (\vec{x}\cdot\vec{u})\vec{u}$, by Definition 2.2.1.

**Ch 5.TF.10** T, since $(7A)^T = 7A^T = 7A$.

**Ch 5.TF.11** F. The Pythagorean Theorem holds for orthogonal vectors $\vec{x}, \vec{y}$ only (Theorem 5.1.9)

**Ch 5.TF.12** T. $\det\begin{bmatrix} a & c \\ b & d \end{bmatrix} = ad - bc = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

**Ch 5.TF.13** T. If $A$ is orthogonal, then $A^T = A^{-1}$, and $A^{-1}$ is orthogonal by Theorem 5.3.4b.

**Ch 5.TF.14** F. Consider $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

**Ch 5.TF.15** F. Consider $A = B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Then $AB^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ isn't equal to $B^TA = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

**Ch 5.TF.16** F. It is required that the columns of $A$ be orthonormal (Theorem 5.3.10). As a counterexample, consider $A = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, with $AA^T = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$.

**Ch 5.TF.17** T, since $(ABBA)^T = A^TB^TB^TA^T = ABBA$, by Theorem 5.3.9a

**Ch 5.TF.18** T, since $A^TB^T = (BA)^T = (AB)^T = B^TA^T$, by Theorem 5.3.9a

**Ch 5.TF.19** F. $\dim(V) + \dim(V^\perp) = 5$, by Theorem 5.1.8c. Thus one of the dimensions is even and the other odd.

**Ch 5.TF.20** T. Consider the $QR$ factorization (Theorem 5.2.2)

**Ch 5.TF.21** F. $\det\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = -1 - 0 = -1$, yet $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is orthogonal.

**Ch 5.TF.22** T. $\left[\frac{1}{2}(A - A^T)\right]^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - A) = -\left[\frac{1}{2}(A - A^T)\right]$.

**Ch 5.TF.23** T, since the columns are unit vectors.

**Ch 5.TF.24** T. Use the Gram-Schmidt process to construct such a basis (Theorem 5.2.1)

**Ch 5.TF.25** F. The columns fail to be unit vectors (use Theorem 5.3.3b)

**Ch 5.TF.26** T, by definition of an orthogonal projection (Theorem 5.1.4).

**Ch 5.TF.27** F. As a counterexample, consider $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

**Ch 5.TF.28** T, by Theorem 5.4.1.

**Ch 5.TF.29** T, by Theorem 5.4.2a.

**Ch 5.TF.30** F. Consider $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, or any other symmetric matrix that fails to be orthogonal.

**Ch 5.TF.31** T. Try $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, so that $A + B = \begin{bmatrix} 1+\cos\theta & -\sin\theta \\ \sin\theta & 1+\cos\theta \end{bmatrix}$. It is required that $\begin{bmatrix} 1+\cos\theta \\ \sin\theta \end{bmatrix}$ and $\begin{bmatrix} -\sin\theta \\ 1+\cos\theta \end{bmatrix}$ be unit vectors, meaning that $1 + 2\cos\theta + \cos^2\theta + \sin^2\theta = 2 + 2\cos\theta = 1$, or $\cos\theta = -\frac{1}{2}$, and $\sin\theta = \pm\frac{\sqrt{3}}{2}$. Thus $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ is a solution.

**Ch 5.TF.32** F. Consider $A = \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$, for example, representing a rotation combined with a scaling.

**Ch 5.TF.33** F. Consider $A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$.

**Ch 5.TF.34** T. By Definition 5.1.12, quantity $\cos(\theta) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$ is positive, so that $\theta$ is an acute angle.

**Ch 5.TF.35** T. In Theorem 5.4.1, let $A = B^T$ to see that $(\text{im}(B^T))^{\perp} = \ker(B)$. Now take the orthogonal complements of both sides and use Theorem 5.1.8d.

**Ch 5.TF.36** T, since $(A^TA)^T = A^T(A^T)^T = A^TA$, by Theorem 5.3.9a.

**Ch 5.TF.37** F. Verify that matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ are similar.

**Ch 5.TF.38** F. Consider $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. The correct formula $\text{im}(B) = \text{im}(BB^T)$ follows from Theorems 5.4.1 and 5.4.2.

**Ch 5.TF.39** T. We know that $A^T = A$ and $S^{-1} = S^T$. Now $(S^{-1}AS)^T = S^TA^T(S^{-1})^T = S^{-1}AS$, by Theorem 5.3.9a.

**Ch 5.TF.40** T. By Theorem 5.4.2, we have $\ker(A) = \ker(A^TA)$. Replacing $A$ by $A^T$ in this formula, we find that $\ker(A^T) = \ker(AA^T)$. Now $\ker(A) = \ker(A^TA) = \ker(AA^T) = \ker(A^T)$.

**Ch 5.TF.41** T. We attempt to write $A = S + Q$, where $S$ is symmetric and $Q$ is skew-symmetric. Then $A^T = S^T + Q^T = S - Q$. Adding the equations $A = S + Q$ and $A^T = S - Q$ together gives $2S = A + A^T$ and $S = \frac{1}{2}(A + A^T)$. Similarly we find $Q = \frac{1}{2}(A - A^T)$. Check that the decomposition $A = S + Q = (\frac{1}{2}(A + A^T)) + (\frac{1}{2}(A - A^T))$ does the job.

**Ch 5.TF.42** T. Apply the Cauchy-Schwarz inequality (squared), $(\vec{x}\cdot\vec{y})^2 \leq \|\vec{x}\|^2\|\vec{y}\|^2$, to $\vec{x} = \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix}$ (all $n$ entries are 1).

**Ch 5.TF.43** T. Let $A = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$. We know that $AA^T = A^2$, or $\begin{bmatrix} x^2+y^2 & xz+yt \\ xz+yt & z^2+t^2 \end{bmatrix} = \begin{bmatrix} x^2+yz & xy+yt \\ zx+tz & yz+t^2 \end{bmatrix}$. We need to show that $y = z$. If $y \neq 0$, this follows from the equation $x^2 + y^2 = x^2 + yz$; if $z \neq 0$, it follows from $z^2 + t^2 = yz + t^2$; if both $y$ and $z$ are zero, we are all set.

**Ch 5.TF.44** T, since $\vec{x}\cdot(\text{proj}_V\vec{x}) = (\text{proj}_V\vec{x} + (\vec{x} - \text{proj}_V\vec{x}))\cdot\text{proj}_V\vec{x} = \|\text{proj}_V\vec{x}\|^2 \geq 0$. Note that $\vec{x} - \text{proj}_V\vec{x}$ is orthogonal to $\text{proj}_V\vec{x}$, by the definition of a projection.

**Ch 5.TF.45** T. Note that $1 = \left\|A\left(\frac{1}{\|\vec{x}\|}\vec{x}\right)\right\| = \left\|\frac{1}{\|\vec{x}\|}A\vec{x}\right\| = \frac{1}{\|\vec{x}\|}\|A\vec{x}\|$ for all nonzero $\vec{x}$, so that $\|A\vec{x}\| = \|\vec{x}\|$. See Definition 5.3.1.

**Ch 5.TF.46** T. If $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ is a symmetric matrix, then $A - xI_2 = \begin{bmatrix} a-x & b \\ b & c-x \end{bmatrix}$. This matrix fails to be invertible if (and only if) $\det(A - xI_2) = (a-x)(c-x) - b^2 = 0$. We use the quadratic formula to find the (real) solutions $x = \frac{a+c\pm\sqrt{(a+c)^2-4ac+4b^2}}{2} = \frac{a+c\pm\sqrt{(a-c)^2+4b^2}}{2}$. Note that the discriminant $(a-c)^2+4b^2$ is positive or zero.

**Ch 5.TF.47** T; one basis is: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

**Ch 5.TF.48** F; A direct computation or a geometrical argument shows that $Q = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, representing a reflection, not a rotation.

**Ch 5.TF.49** F; $\dim(\mathbb{R}^{3\times 3}) = 9$, $\dim(\mathbb{R}^{2\times 2}) = 4$, so $\dim(\ker(L)) \geq 5$, but the space of all $3\times 3$ skew-symmetric matrices has dimension of 3.

$$\left(\text{A basis is } \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.\right)$$

**Ch 5.TF.50** T; Consider an orthonormal basis $\vec{v}_1, \vec{v}_2$ of $V$, and a unit vector $\vec{v}_3$ perpendicular to $V$, and form the orthogonal matrix $S = [\vec{v}_1\ \vec{v}_2\ \vec{v}_3]$. Now $AS = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{0} \end{bmatrix} = S\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Since $S$ is orthogonal, we have $S^TAS = S^{-1}AS = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, a diagonal matrix.