Complex Numbers (7.8) **Math 098**

Definition: The number *i*

i is the unique number for which $i = \sqrt{-1}$ and $i^2 = -1$

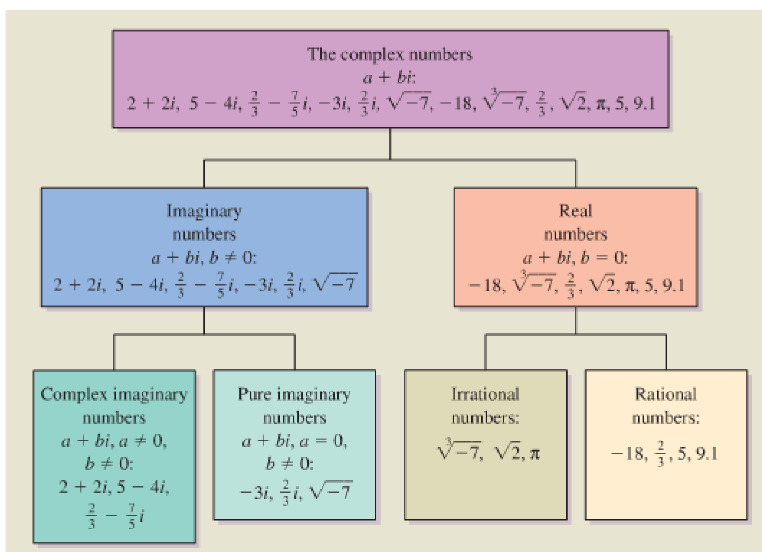
We can now define the root $\sqrt{-a} = \sqrt{-1}\sqrt{a} = i\sqrt{a}$ provided *a* is non-negative.

Warning: $i \neq$ ______________________

Example 1: Express in terms of *i*.

a.) $\sqrt{-15}$ b.) $\sqrt{-9}$ c.) $-\sqrt{-50}$

Definition: Imaginary numbers
An *imaginary number* is a number that can be written in the form $a + bi$, where *a* and *b* are real numbers and $b \neq 0$.



Imaginary numbers have many real world applications in engineering and the physical sciences. Some applications include: control theory, improper integrals, fluid dynamics, dynamic equations, electromagnetism and electrical engineering, signal analysis, quantum mechanics, relativity, geometry, fractals, algebraic number theory, and analytic number theory

Note: Imaginary numbers are sometimes called complex numbers.

Example 2: Add or subtract

a.) $(4-5i)+(2+3i)$

b.) $(3-i)-(5-2i)$

Warning: $\sqrt{-3}\cdot\sqrt{-3}$

Example 3: Multiply and simplify. Write you answers in the standard $a+bi$ form

a.) $\sqrt{-9}\cdot\sqrt{-36}$

b.) $\sqrt{-6}\cdot\sqrt{-10}$

c.) $-2i\cdot 7i$

d.) $3i(4-7i)$

e.) $(2-3i)(4+5i)$

f.) $(3-5i)^2$

<u>Definition</u>: Conjugate of a complex number

The *conjugate* of a complex number $a+bi$ is $a-bi$ and the conjugate of $a-bi$ is $a+bi$.

Example 4: Find and multiply by the conjugate

a.) $-2+5i$

conjugate: ___________ and the product:

b.) $3-7i$

conjugate: ___________ and the product:

c.) $5i$

conjugate: ___________ and the product:

<u>Method</u>: When dividing by complex numbers, we multiply by the ______________________

______________________ as a ______________________ in a manner similar to how we

rationalize the denominator.

Example 5: Divide. Write your answers in the form $a+bi$

a.) $\dfrac{4}{2-3i}$

b.) $\dfrac{2+7i}{5i}$

Explore powers of *i*

i =

i^2 =

i^3 =

i^4 =

i^5 =

i^6 =

i^7 =

i^8 =

Divide powers by ___________

Remainder	Result

Example 6: Simplify

a.) i^{28}

b.) i^{46}

c.) i^{33}

d.) i^{75}

You can also work with complex numbers on the graphing calculator …