Polynomial and Rational Inequalities (8.9) **Math 098**

Overview: Quadratic inequalities

Consider the graph of $f(x) = x^2 - 3x - 10$

a.) When is $y = 0$? (x-intercepts)

$x = -2$ or $x = 5$

b.) When is $y < 0$? (below x-axis)

$-2 < x < 5$ $\quad (-2, 5)$

c.) When is $y > 0$? (above x-axis)

$x < -2$ OR $5 < x$ $\quad (-\infty, -2) \cup (5, \infty)$

Look at it another way: notice that $x^2 - 3x - 10 = (x-5)(x+2) = 0$ divides the graph into three regions.

Sign Diagram

y-vals: $+ \quad 0 \quad - \quad 0 \quad +$ → x

x-vals: -2, 5

Test -3 Test 0 Test 6

Table

Test Points	$(x-5)(x+2)$
-3	$(-3-5)(-3+2) = 8\ (+)$
0	$(-5)(2) = -10\ (-)$
6	$(6-5)(6+2) = 8\ (+)$

Example 1: Solve $x^2 + 2x - 8 > 0$ (positive)

$\Rightarrow (x-2)(x+4) > 0$

$+ \quad 0 \quad - \quad 0 \quad +$ → x

-4, 2

test -5 test 0 test 3

$x < -4$ OR $2 < x$

$(-\infty, -4) \cup (2, \infty)$

Example 2: Solve $(x+3)(x-4)(x-1) \le 0$ (zero or negative)

y vals: $- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$ x

$-3 \qquad 1 \qquad 4$

x vals: TEST: $-4 \qquad 0 \qquad 2 \qquad 5$

$(-\infty, -3] \cup [1, 4]$

$x \le -3$ OR $1 \le x \le 4$

Example 3: $x^4 - x^3 - 12x^2 < 0$ (negative)

$\Rightarrow x^2(x^2 - x - 12) < 0$

$\Rightarrow x^2(x+3)(x-4) < 0$

$+ \quad 0 \quad - \quad 0 \quad - \quad 0 \quad +$ x

$-3 \qquad 0 \qquad 4$

$-4 \qquad -1 \qquad 1 \qquad 5$

$(-)^2(-)(-) \qquad (-)^2(+)(-) \qquad (+)^2(+)(-) \qquad (+)^2(+)(+)$

$-3 < x < 0$ or $0 < x < 4$

$(-3, 0) \cup (0, 4)$

<u>Method</u>: To solve a polynomial inequality

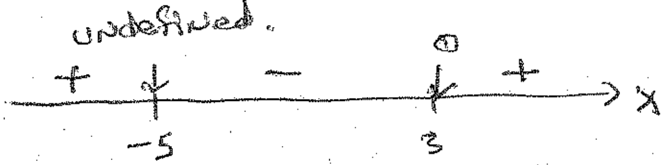
a.) Add or subtract to get 0 on one side and solve the related polynomial equation $p(x) = 0$

b.) Use the numbers found in step (a.) to divide the number line into intervals

c.) Using a test value from each interval or the graph of the related function, determine the sign of $p(x)$ over each interval

d.) Select the interval(s) for which the inequality is satisfied and write set notation or interval notation for the solution set. Include the endpoints of the intervals when $\le$ or $\ge$ is used

Example 4: Solve $\frac{x-3}{x+5} \le 0$ (zero or negative)



-6 $\qquad$ 0 $\qquad$ 4

$\frac{(-)}{(-)}$ $\qquad$ $\frac{(-)}{(+)}$ $\qquad$ $\frac{(+)}{(+)}$

$-5 < x \le 3$ inequality

$(-5, 3]$ interval notation.

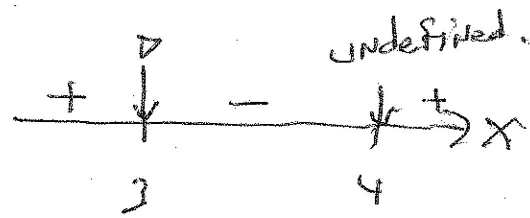
Example 5: Solve $\frac{x-1}{x-4} > -2$ (positive)

$$\Rightarrow \frac{x-1}{x-4} + 2 > 0$$

$$\Rightarrow \frac{x-1+2(x-4)}{x-4} > 0$$

$$\Rightarrow \frac{3x-9}{x-4} > 0$$

$$\Rightarrow \frac{3(x-3)}{x-4} > 0$$



$\frac{(-)}{(-)}$ $\qquad$ $\frac{(+)}{(-)}$ $\qquad$ $\frac{(+)}{(+)}$

$x < 3$ OR $4 < x$

$(-\infty, 3) \cup (4, \infty)$

Method: To solve a rational inequality

a.) Change the inequality symbol to an equals sign and solve the related equation $r(x)=0$

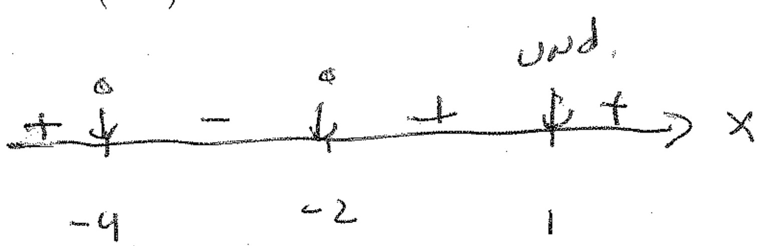
b.) Find any replacements for which the rational expression is undefined

c.) Use the numbers found in steps (a.) and (b.) to divide the number line into intervals

d.) Using a test value from each interval or the graph of the related function, determine the sign of $r(x)$ over each interval

e.) Select the interval(s) for which the inequality is satisfied and write set notation or interval notation for the solution set. If the inequality symbol is $\leq$ or $\geq$, then the solutions from (a.) are also included in the solution set. Those numbers found in (b.) should be excluded from the solution set even if they are solutions from (a.)

Example 6: Solve $\dfrac{(x+2)(x+4)}{(x-1)^2} \geq 0$ (positive or zero)



$\dfrac{(-)(-)}{(+)}$ $\dfrac{(-)(+)}{(+)}$ $\dfrac{(+)(+)}{(+)}$ $\dfrac{(+)(+)}{(+)}$

$x \leq -4$ OR $-2 \leq x < 1$ OR $1 < x$

$(-\infty, -4] \cup [-2, 1) \cup (1, \infty)$