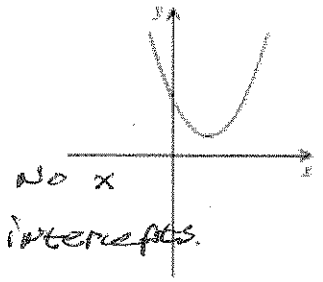
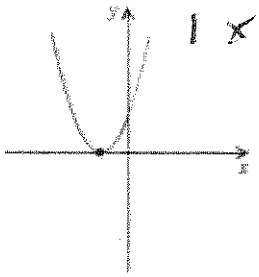
Studying Quadratic Solutions (8.3) **Math 098**

Recall: The solutions to $ax^2+bx+c=0$ for $a \neq 0$ are given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

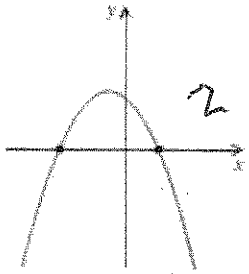
The Discriminant: b^2-4ac



The discriminant <0



The discriminant $=0$



The discriminant >0

Example 1: Consider the following quadratic equations. How many and what type of solutions are there in each of these examples.

a.) $x^2-2x+4=0$

Discriminant $= 4-4(1)(4) = -12$

No x-intercepts.

2 complex zeros.

b.) $4x^2-12x+9=0$

Discriminant $= 144-4(4)(9) = 0$

1 real x-intercept.

c.) $6x^2+5x-4=0$

Discriminant $= 25-4(6)(-4) = 121$

2 x-int. (real).

Example 2: Find quadratic equation(s) for which the given values are the solutions

a.) 4 and 2/3

$\hookrightarrow (x-4)(x-\frac{2}{3}) = 0$

or $(x-4)(3x-2) = 0 \longrightarrow 3x^2 - 14x + 8 = 0$ IN STANDARD FORM

or $c(x-4)(3x-2) = 0$, $c \neq 0$.

b.) -5

$\hookrightarrow (x+5)^2 = 0 \longrightarrow x^2 + 10x + 25 = 0$ standard form

$c(x+5)^2 = 0$, $c \neq 0$

c.) $3\sqrt{2}$

$\hookrightarrow (x-3\sqrt{2})^2 = 0$ *

or $c(x-3\sqrt{2})^2 = 0$, $c \neq 0$.

* in standard form

$x^2 - 6\sqrt{2}x + 18 = 0$

d.) $5-2i$ and $5+2i$ (solve using two methods)

method 1: $(x-(5-2i))(x-(5+2i))=0$

$\Rightarrow x^2-(5+2i)x-(5-2i)x+(5-2i)(5+2i)=0$

$\Rightarrow x^2-5x-2ix-3x+2ix+25+10i-10i-4i^2=0$ ($-4i^2=+4$)

$\Rightarrow x^2-10x+29=0$

method 2: $x=5\pm 2i$

$\Rightarrow x-5=\pm 2i$

$\Rightarrow (x-5)^2=(\pm 2i)^2$

$\Rightarrow x^2-10x+25=-4$

$\Rightarrow x^2-10x+29=0$

e.) $\frac{5}{4}+\frac{\sqrt{33}}{4}$ and $\frac{5}{4}-\frac{\sqrt{33}}{4}$ (solve using two methods)

method 2: $x=\frac{5}{4}\pm\frac{\sqrt{33}}{4}$

$\Rightarrow x-\frac{5}{4}=\pm\frac{\sqrt{33}}{4}$

$\Rightarrow \left(x-\frac{5}{4}\right)^2=\left(\pm\frac{\sqrt{33}}{4}\right)^2$

$\Rightarrow x^2-\frac{5}{2}x+\frac{25}{16}=\frac{33}{16}$

$\Rightarrow x^2-\frac{5}{2}x-\frac{8}{16}=0$

$\Rightarrow x^2-\frac{5}{2}x-\frac{1}{2}=0$

method 1: $\left(x-\left(\frac{5}{4}+\frac{\sqrt{33}}{4}\right)\right)\left(x-\left(\frac{5}{4}-\frac{\sqrt{33}}{4}\right)\right)=0$

$\Rightarrow x^2-\left(\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x-\left(\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x+\left(\frac{5}{4}-\frac{\sqrt{33}}{4}\right)\left(\frac{5}{4}+\frac{\sqrt{33}}{4}\right)=0$

$\Rightarrow x^2-\frac{5}{2}x+\left(\frac{25}{16}-\frac{33}{16}\right)=0$

$\Rightarrow x^2-\frac{5}{2}x-\frac{1}{2}=0$