Expressions With Several Radicals (7.5) **Math 098**

Like radicals have the same index and radicand. These can be combined similarly to "like terms" of variables.

Example 1: Simplify by combining like radicals

a.) $3\sqrt{5}+5\sqrt{5}$

$= 8\sqrt{5}$

b.) $\sqrt[3]{3}-5x\sqrt[3]{3}+7\sqrt[3]{3}$

$=(1-5x+7)\sqrt[3]{3}$

$=(8-5x)\sqrt[3]{3}$

c.) $3\sqrt{2}+4\sqrt{3}-\sqrt{2}-7\sqrt{3}+\sqrt[3]{2}$

$2\sqrt{2} - 3\sqrt{3} + \sqrt[3]{2}$

d.) $4\sqrt{8}-6\sqrt{2}$

$= 4\sqrt{2\cdot 2\cdot 2} - 6\sqrt{2}$

$= 4\cdot 2\sqrt{2} - 6\sqrt{2}$

$= 8\sqrt{2} - 6\sqrt{2}$

$= 2\sqrt{2}$

e.) $3\sqrt[3]{16}+\sqrt[3]{54}$

$= 3\sqrt[3]{8\cdot 2} + \sqrt[3]{27\cdot 2}$

$= 3\cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$

$= 6\sqrt[3]{2} + 3\sqrt[3]{2}$

$= 9\sqrt[3]{2}$

Example 2: Multiply

a.) $\sqrt{7}\left(3-\sqrt{7}\right)$

$= 3\sqrt{7} - \sqrt{7}\cdot\sqrt{7}$

$= 3\sqrt{7} - 7$

b.) $\sqrt[3]{2}\left(\sqrt[3]{4}-2\sqrt[3]{32}\right)$

$= \sqrt[3]{8} - 2\sqrt[3]{64}$

$= 2 - 2(4)$

$= -6$

c.) $\left(2\sqrt{3}-4\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$

$= 2\sqrt{9} + 2\sqrt{6} - 4\sqrt{6} - 4\sqrt{4}$

$= 6 - 2\sqrt{6} - 8$

$= -2 - 2\sqrt{6}$

d.) $\left(4-\sqrt{5}\right)^2$

$= (4-\sqrt{5})(4-\sqrt{5})$

$= 16 - 4\sqrt{5} - 4\sqrt{5} + 5$

$= 21 - 8\sqrt{5}$

conjugates

e.) $\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)$

$= 9 \cancel{+3\sqrt{7}} \cancel{-3\sqrt{7}} - 7$

$= 2$

Review: Rationalizing the Denominator

a.) $\frac{3}{4-\sqrt{7}} \cdot \frac{4+\sqrt{7}}{4+\sqrt{7}}$

$= \frac{12+3\sqrt{7}}{16-7}$

$= \frac{3(4+\sqrt{7})}{9}$

$= \frac{1}{3}(4+\sqrt{7})$

b.) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{35}-\sqrt{14}+5-\sqrt{10}}{5-2}$

$= \frac{\sqrt{35}-\sqrt{14}+5-\sqrt{10}}{3}$

Method: To simplify products or quotients with differing indices

1.) Convert all radical expressions to exponential notation.
2.) When the bases are identical, subtract exponents to divide and add exponents to multiply. This may require finding a common denominator.
3.) Convert back to radical notation and, if possible, simplify.

Example 3: Simplify (assume variables are positive)

a.) $\sqrt[3]{x^2} \cdot \sqrt[6]{x^5}$

$= x^{\frac{2}{3}} x^{\frac{5}{6}}$

$= x^{\frac{2}{3}+\frac{5}{6}}$

$= x^{3/2}$

$\sqrt{x^3}$

$= x\sqrt{x}$

b.) $\sqrt[5]{a^3 b} \cdot \sqrt{ab}$

$= a^{\frac{3}{5}} b^{\frac{1}{5}} a^{\frac{1}{2}} b^{\frac{1}{2}}$

$= a^{\frac{3}{5}+\frac{1}{2}} b^{\frac{1}{5}+\frac{1}{2}}$

$= a^{\frac{11}{10}} b^{\frac{7}{10}}$

$= a\sqrt[10]{ab^7}$

Example 4: Simplify $\dfrac{\sqrt[3]{(2+5x)^2}}{\sqrt[4]{2+5x}}$ (assume variables are positive)

$$= \frac{(2+5x)^{2/3}}{(2+5x)^{1/4}}$$

$$= (2+5x)^{\frac{2}{3}-\frac{1}{4}} \longleftarrow \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$

$$= \sqrt[12]{(2+5x)^5}$$

Example 5: Find $(f \cdot g)(x)$ if $f(x) = \sqrt[4]{x^7} + \sqrt[4]{3x^2}$ and $g(x) = \sqrt[4]{x}$

$$f \cdot g = \left(\sqrt[4]{x^7} + \sqrt[4]{3x^2}\right) \cdot \sqrt[4]{x}$$

$$= \sqrt[4]{x^8} + \sqrt[4]{3x^3}$$

$$= x^2 + \sqrt[4]{3x^3}$$

Example 6: Let $f(x) = x^2$. Find $f\left(\sqrt{6} - \sqrt{3}\right)$

$$\Rightarrow f(\sqrt{6} - \sqrt{3}) = (\sqrt{6} - \sqrt{3})^2$$

$$= (\sqrt{6} - \sqrt{3})(\sqrt{6} - \sqrt{3})$$

$$= 6 - \sqrt{18} - \sqrt{18} + 3$$

$$= 9 - 2\sqrt{18} \quad \leftarrow 9 \cdot 2$$

$$= 9 - 6\sqrt{2}$$