

Method: (The quotient rule for radicals) For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}, b \neq 0$, we have $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example 1: Simplify

$$\begin{aligned} \text{a.) } \sqrt{\frac{144}{81}} &= \frac{\sqrt{144}}{\sqrt{81}} \\ &= \frac{12}{9} \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{b.) } \sqrt[3]{\frac{125}{216}} &= \frac{\sqrt[3]{125}}{\sqrt[3]{216}} \\ &= \frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{c.) } \sqrt{\frac{25x^5}{y^6}} &= \frac{5x^2}{y^3} \sqrt{x} \\ &= \frac{5x^2\sqrt{x}}{y^3} \end{aligned}$$

$$\text{d.) } \sqrt[3]{\frac{8a^8}{27b^{14}}} = \frac{2a^2}{3b^4} \sqrt[3]{\frac{a^2}{b^2}}$$

$$\begin{aligned} \text{e.) } \frac{\sqrt{50}}{\sqrt{2}} &= \sqrt{\frac{50}{2}} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{f.) } \frac{\sqrt[3]{81}}{\sqrt[3]{3}} &= \sqrt[3]{\frac{81}{3}} \\ &= \sqrt[3]{27} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{g.) } \frac{\sqrt{72xy}}{2\sqrt{2}} &= \frac{1}{2} \frac{\sqrt{72xy}}{\sqrt{2}} \\ &= \frac{1}{2} \sqrt{\frac{72xy}{2}} \\ &= \frac{1}{2} \sqrt{36xy} \\ &= \frac{6}{2} \sqrt{xy} \\ &= 3\sqrt{xy} \end{aligned}$$

$$\begin{aligned} \text{h.) } \frac{\sqrt[5]{64a^{11}b^{28}}}{\sqrt[3]{2ab^{-2}}} &= \sqrt[5]{\frac{64a^{11}b^{28}}{2ab^{-2}}} \\ &= \sqrt[5]{32a^{10}b^{30}} \\ &= 2a^2b^6 \end{aligned}$$

Concept: Rationalizing the Denominator

How might we add $\sqrt{\frac{1}{3}}$ and $\frac{1}{3}$?

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{and} \quad \frac{1}{3} \quad \text{LCD} = 3$$
$$\frac{\sqrt{3}}{3}$$

Example 2: Rationalize the denominator

a.) $\frac{\sqrt{5}}{\sqrt{11}}$

$$= \frac{\sqrt{5}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}}$$

$$= \frac{\sqrt{55}}{11}$$

b.) $\frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$$= \frac{3\sqrt{7}}{7}$$

c.) $\frac{2}{\sqrt{5}-3}$

multiply by the conjugate.

$$= \frac{2}{\sqrt{5}-3} \cdot \frac{\sqrt{5}+3}{\sqrt{5}+3}$$

$$= \frac{2\sqrt{5}+6}{5+3\sqrt{5}-3\sqrt{5}-9}$$

$$= \frac{2\sqrt{5}+6}{-4}$$

$$= -\frac{\sqrt{5}+3}{2}$$

✓
 $\sqrt{5}-3$ and $\sqrt{5}+3$

$2-\sqrt{7}$ and $2+\sqrt{7}$

$\sqrt{13}+1$ and $\sqrt{13}-1$

d.) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{3+\sqrt{6}-\sqrt{6}-2}$$

$$= (\sqrt{3}+\sqrt{2})^2$$