Special Cases: Squares and Cubes (5.6 & 5.7) **Math 098**

Review: Multiply $(x+5)^2$ $= x^2 + 10x + 25$

$x = A$

$5 = B$

The formulas: Factoring a perfect-square trinomial

- $A^2 + 2AB + B^2 = (A+B)^2$
- $A^2 - 2AB + B^2 = (A-B)^2$

To recognize a perfect-square trinomial if given $ax^2 + bx + c$

- Are *a* and *c* perfect squares? If they are, of what are they squares? These are your *A* and *B*.
- Is the middle term of the form $2AB$

Example 1: Factor

a.) $x^2 - 14x + 49$

$= (x-7)(x-7)$

$= (x-7)^2$

$A = x$

$B = 7$

b.) $9r^2 + 36rs + 36s^2$ $= 9(r^2+4rs+4s^2)$

$= (3r+6s)(3r+6s)$

$= (3r+6s)^2$

$= 9(r+2s)^2$

$A = 3r$

$B = 6s$

c.) $25\alpha^2 - 25\alpha + 4$ ← Not a perfect square

$\neq (5\alpha-2)(5\alpha-2)$

$= (5\alpha-4)(5\alpha-1)$

$A = 5\alpha$

$B = 2$

20

5	-4	1	2
5	-1	4	2

5

d.) $25x^2 - 20x + 4$

$= (5x-2)(5x-2)$

$A = 5x$

$B = 2$

Example 2: Factor

a.) $8n^2 - 40n + 50$

$= 2(4n^2 - 20n + 25)$

$= 2(2n - 5)^2$

b.) $-4y^2 - 144y^8 + 48y^5$

$= -144y^8 + 48y^5 - 4y^2$

$= -4y^2(36y^6 - 12y^3 + 1)$

$= -4y^2(6y^3 - 1)^2$

The formula: <u>Factoring a difference of two squares</u>

- $A^2 - B^2 = (A+B)(A-B)$

To factor a difference of two squares, write the product of the sum and difference of the quantities being squared. *The sum of squares cannot be factored.*

Example 3: Factor

a.) $x^2 - 81$

$= (x)^2 - 9^2$

$= (x-9)(x+9)$

b.) $16a^4 - 25b^2$

$= (4a^2)^2 - (5b)^2$

$= (4a^2 - 5b)(4a^2 + 5b)$

Example 4: Solve

a.) $16 = 81r^4$

$\Rightarrow 0 = 81r^4 - 16$

$\Rightarrow (9r^2 + 4)(9r^2 - 4) = 0$

$\Rightarrow (9r^2 + 4)(3r+2)(3r-2) = 0$

$\Rightarrow 9r^2 + 4 = 0$ or $3r + 2 = 0$

or $3r - 2 = 0$

$\Rightarrow r = \pm \frac{2}{3}$

b.) $x^3 + 3x^2 = 9x + 27$

$\Rightarrow x^3 + 3x^2 - 9x - 27 = 0$

$\Rightarrow (x^3 + 3x^2) - (9x + 27) = 0$

$\Rightarrow x^2(x+3) - 9(x+3) = 0$

$\Rightarrow (x+3)(x^2 - 9) = 0$

$\Rightarrow (x+3)(x+3)(x-3) = 0$

$\Rightarrow x = \pm 3$

Review: Multiply $(A+B)(A^2-AB+B^2)$

$$= A^3 - \cancel{A^2B} + \cancel{AB^2} + \cancel{A^2B} - \cancel{AB^2} + B^3$$

$$= A^3 + B^3$$

The formulas: Factoring a sum or a difference of two cubes

- $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
- $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Example 5: Factor

a.) $x^3 + 27$

$= (x)^3 + 3^3$

$= (x+3)(x^2 - 3x + 9)$

b.) $125a^3 - 216b^3$

$= (5a)^3 - (6b)^3$

$= (5a - 6b)(25a^2 + 30ab + 36b^2)$

c.) $r^6 - 64$

$= (r^2)^3 - (4)^3$

$= (r^2 - 4)(r^4 + 4r^2 + 16)$

$= (r+2)(r-2)(r^4 + 4r^2 + 16)$

d.) $2y^4 - 16y$

$= 2y(y^3 - \overset{2^3}{8})$

$= 2y(y-2)(y^2 + 2y + 4)$

Summary: Useful factoring facts

Factoring a perfect-square trinomial

- $A^2 + 2AB + B^2 = (A+B)^2$ or $A^2 - 2AB + B^2 = (A-B)^2$

Factoring a difference of two squares

- $A^2 - B^2 = (A+B)(A-B)$

Factoring a sum or a difference of two cubes

- $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$ or $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$