

Let's investigate patterns from factoring that we have learned to date:

a.) $x^2 + 10x + 16$

$$= (x+8)(x+2)$$

b.) $x^2 + 6x - 16$

$$= (x+8)(x-2)$$

c.) $x^2 - 10x + 16$

$$= (x-8)(x+2)$$

d.) $x^2 - 6x - 16$

$$= (x-8)(x+2)$$

In review:

$$a, b, c > 0$$

$$ax^2 \pm bx + c$$

same sign

$$\begin{array}{cc} \pm b & \begin{array}{cc} \rightarrow & + & + \\ \rightarrow & - & - \end{array} \end{array}$$

$$ax^2 \pm bx - c$$

opposite signs

$$\begin{array}{cc} + & - \\ - & + \end{array} \left. \vphantom{\begin{array}{cc} + & - \\ - & + \end{array}} \right\} \text{careful.}$$

and multiply $(2x+1)(x+6) = 2x^2 + 12x + x + 6$

$$= 2x^2 + 13x + 6$$

and multiply $(2x+3)(3x-4) = 6x^2 - 8x + 9x - 12$

$$= 6x^2 + x - 12$$

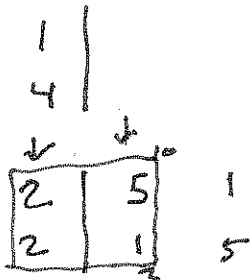
Go back to the previous two questions and consider how many possibilities must be considered if we guess and check.

1st - 4 combos
2nd - 24 "

For the following "method" to work, we MUST ALWAYS factor out the GCF first.

Example 1: Factor

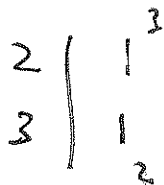
a.) $4x^2 + 12x + 5 = (2x + 5)(2x + 1)$



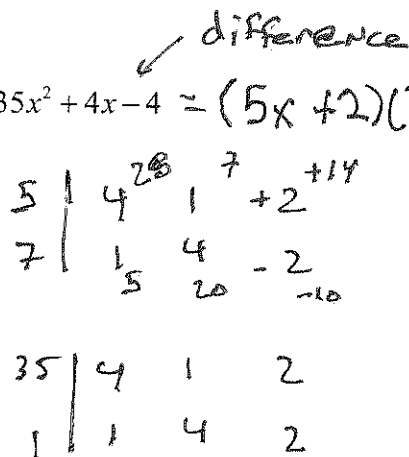
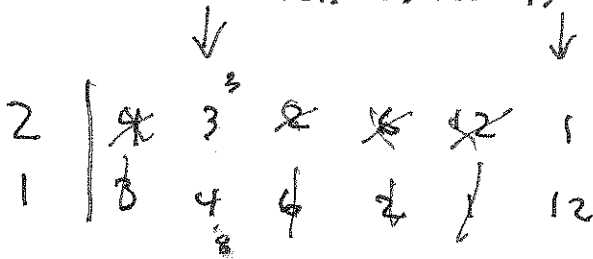
check: $4x^2 + 2x + 10x + 5 \checkmark$

Hint: When you can, start with the most "middle" values for a).

b.) $6x^2 - 5x + 1 = (2x - 1)(3x - 1)$



c.) $2x^2 - 11x + 12 = (2x - 3)(x - 4)$ d.) $35x^2 + 4x - 4 = (5x + 2)(7x - 2)$



$$e.) 14a^2 - 3ab - 2b^2$$

$$= (7a + 2b)(2a - b)$$

consider difference

$$14a^2 - 3a - 2$$

$$+4$$

$$\begin{array}{r|rr} 7 & +2 & 1 \\ 2 & -1 & 2 \\ & -7 & \end{array}$$

$$\begin{array}{r|rr} 1 & 2 & 1 \\ 14 & 1 & 2 \end{array}$$

$$g.) -5x^2 - 19x + 4$$

$$= -(5x^2 + 19x - 4)$$

$$= -(5x - 1)(x + 4)$$

$$\begin{array}{r|rr} 5 & 2 & -1 & 4 & \rightarrow 5x - 1 \\ 1 & 2 & +4 & 1 & \rightarrow x + 4 \\ & 10 & +20 & \end{array}$$

$$i.) 42a^2b + 55ab - 25b$$

$$= b(42a - 5)(3a + 5)$$

$$42a^2 + 55a - 25$$

$$\begin{array}{r|rr} 6 & 1 & 25 & 5 & 35 \\ 7 & 25 & 1 & 5 & 30 \\ & 25 & 1 & 5 & \end{array}$$

$$\begin{array}{r|rr} 14 & 1 & 25 & -5 & -15 \\ 3 & 25 & 1 & +5 & +70 \\ & 25 & 1 & +5 & +70 \end{array}$$

$$b(14a - 5)(3a + 5)$$

$$f.) 6xy^2 + 33xy - 18x$$

$$= 3x(2y^2 + 11y - 6)$$

$$= 3x(2y - 1)(y + 6)$$

$$\begin{array}{r|rr} 2 & 2 & 3 & -1 & 6 \\ 1 & 2 & 3 & -1 & 6 \\ & 2 & 6 & 9 & +12 \end{array}$$

$$h.) 30x^2 - 23x - 45$$

$$\begin{array}{r|rr} 10 & 1 & 45 & 9 & +27 & 15 \\ 3 & 15 & 1 & -5 & 3 & 15 \\ & 15 & 1 & -5 & 3 & 15 \end{array}$$

$$= (10x + 9)(3x - 5)$$

check:

$$30x^2 - 50x + 27x - 45$$

$$j.) 12x^2 + 5x - 6$$

$$\begin{array}{r|rr} 12 & 1 & 6 & 2 & 3 \\ 1 & 6 & 1 & 3 & 2 \\ & 6 & 1 & 3 & 2 \end{array}$$

$$\begin{array}{r|rr} 6 & 1 & 2 & 3 \\ 2 & 6 & 1 & 3 \end{array}$$

$$\begin{array}{r|rr} 4 & 1 & 2 & 3 \\ 3 & 6 & 1 & 3 \end{array}$$

Prime.

Important: If the leading coefficient is 1 ($x^2 + bx + c$) then you do NOT need a table.

Example 2: Solve $30x^2 - 26x + 4 = 0$ \hookrightarrow LCF = 2

$$\Rightarrow 15x^2 - 13x + 2 = 0$$

$$\Rightarrow (5x-1)(3x-2) = 0$$

$$\Rightarrow 5x-1=0 \text{ OR } 3x-2=0$$

$$\Rightarrow 5x=1 \text{ OR } 3x=2$$

$$\Rightarrow x = \frac{1}{5} \text{ OR } x = \frac{2}{3}$$

$$\begin{array}{r|rr} 1 & & \\ \hline 15 & 5 & -1 & 2 \\ & 3 & -2 & 1 \\ & & -10 & \end{array}$$

Example 3: Given $f(x) = 24x^2 - 37x$, find a such that $f(a) = 72$.

$$f(a) = 24a^2 - 37a$$

$$\text{solve } 24a^2 - 37a = 72$$

$$\Rightarrow 24a^2 - 37a - 72 = 0$$

$$(3a-8)(8a+9) = 0$$

$$a = \frac{8}{3} \text{ OR } a = -\frac{9}{8}$$

$$\begin{array}{cccccccccccc} 6 & 8 & 9 & 18 & 4 & 1 & 72 & 2 & 2 & 3 & 24 & 6 & 12 \\ 4 & 9 & 8 & 4 & 18 & 72 & 1 & 2 & 2 & 24 & 3 & 12 & 6 \end{array}$$

Example 4: (You try) Factor $-30p^3q + 88p^2q^2 + 6pq^3$

$$= -2pq(15p^2 - 44pq - 3q^2)$$

$$= -2pq(15p+q)(p-3q)$$

$$\begin{array}{r|rr} 3 & -8 & \\ \hline 8 & 49 & \\ & +27 & \end{array}$$

$$\begin{array}{r|rr} 3 & 1 & 3 \\ \hline 5 & 3 & 1 \\ & 1 & 3 \\ & +1 & \\ 15 & +1 & 3 \\ 1 & 3 & 1 \\ & -45 & \end{array}$$

Example 5: Find the x-intercepts of $f(x) = 63x^3 + 111x^2 + 36x$

$$\text{solve } 63x^3 + 111x^2 + 36x = 0$$

$$\Rightarrow 3x(21x^2 + 37x + 12) = 0$$

$$\Rightarrow 3x(7x+3)(3x+4) = 0$$

$$\Rightarrow 3x=0 \text{ OR } 7x+3=0 \text{ OR } 3x+4=0$$

$$\Rightarrow x=0 \text{ OR } x = -\frac{3}{7} \text{ OR } x = -\frac{4}{3}$$

$$\begin{array}{cccc} & 36 & 18 & 9 \\ 7 & 12 & 6 & 3 \\ 3 & 1 & 2 & 4 \\ & 7 & 14 & 28 \end{array}$$