Review

Definitions and Properties of Exponents

The following summary assumes that no denominators are 0 and that 0^0 is not considered. For any integers m and n,

1 as an exponent:	$a^1 = a$
0 as an exponent:	$a^0 = 1$
Negative exponents:	$a^{-n} = \frac{1}{a^n}$
	$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$
	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
The Product Rule:	$a^m \cdot a^n = a^{m+n}$
The Quotient Rule:	$\frac{a^m}{a^n} = a^{m-n}$
The Power Rule:	$(a^m)^n = a^{mn}$
Raising a product to a power:	$(ab)^n = a^n b^n$
Raising a quotient to a power:	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 1: Multiply and simplify

a.) $(3x^3y^8)(-2x^4y^5)$

$= -6x^7y^{13}$

b.) $(-3a^2b^3c^4)(-7a^3b^7c^{11})$

$= 21a^5b^{10}c^{15}$

c.) $3x(4x-7)$

$= 3x(4x) - 3x(7)$

$= 12x^2 - 21x$

d.) $4rs^2(r^2-2s^2)$

$= 4rs^2(r^2) + 4rs^2(-2s^2)$

$= 4r^3s^2 + -8rs^4$

Example 1 continued:

e.) $(x^2-5)(4x^2+3)$

$= x^2(4x^2+3) - 5(4x^2+3)$

$= 4x^4 + 3x^2 - 20x^2 - 15$

$= 4x^4 - 17x^2 - 15$

f.) $(r+3)(r^2-5r+2)$

$= r(r^2-5r+2) + 3(r^2-5r+2)$

$= r^3 - 5r^2 + 2r + 3r^2 - 15r + 6$

$= r^3 - 2r^2 - 13r + 6$

Example 2: Sometimes it can be easier to multiply vertically.

a.) $(3x^2-5x+2)(2x^2+x-4)$

$$\begin{array}{r} 3x^2 - 5x + 2 \\ \times \quad 2x^2 + x - 4 \\ \hline -12x^2 + 20x - 8 \\ 3x^3 - 5x^2 + 2x + 0 \\ 6x^4 - 10x^3 + 4x^2 + 0 + 0 \\ \hline 6x^4 - 7x^3 - 13x^2 + 22x - 8 \end{array}$$

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Example 3: Multiply

a.) $(x+4)(x-3)$

$$= x^2 - 3x + 4x - 12$$

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$$= x^2 + x - 12$$

b.) $(3x-4y)(x-2y)$

$$= 3x^2 - 6xy - 4xy + 8y^2$$

$$= 3x^2 - 10xy + 8y^2$$

c.) $(r-2)\underbrace{(r+3)(r-4)}$

$$= (r-2)(r^2 - 4r + 3r - 12)$$

$$= (r-2)(r^2 - r - 12)$$

$$= r^3 - r^2 - 12r - 2r^2 + 2r + 24$$

$$= r^3 - 3r^2 - 10r + 24$$

Question: Does $(x+4)^2 = x^2 + 16$? Discuss this with your neighbors and figure it out.

$$(x+4)^2 = (x+4)(x+4)$$

$$= x^2 + 4x + 4x + 16$$

$$= x^2 + 8x + 16$$

It's worth memorizing the square of a binomial (*perfect squares*)l:

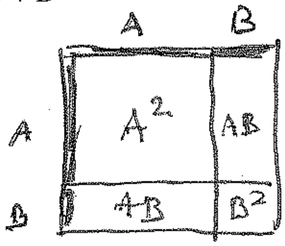
- $(A+B)^2 = A^2 + 2AB + B^2$
- $(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)^2 = (A+B)(A+B)$

$= A^2 + AB + AB + B^2$

$= A^2 + 2AB + B^2$

The picture can help.



Example 4:

a.) $(\overset{A}{x} - \overset{B}{3})^2$

$= x^2 - 2(x)(3) + 3^2$

$= x^2 - 6x + 9$

b.) $(\overset{A}{4x} + \overset{B}{3y})^2$

$= (4x)^2 + 2(4x)(3y) + (3y)^2$

$= 16x^2 + 24xy + 9y^2$

c.) $\left(\overset{A}{5y^3} - \overset{B}{\frac{1}{2}z}\right)^2 = (5y^3)^2 - 2(5y^3)(\frac{1}{2}z) + (\frac{1}{2}z)^2$

$= 25y^6 - 5y^3z + \frac{1}{4}z^2$

perfect squares

Explore the *difference of squares* to find the pattern:

a.) $(x-3)(x+3)$

$= x^2 + \cancel{3x} - \cancel{3x} - 9$

$= x^2 - 9$

b.) $(x+4)(x-4)$

$= x^2 - \cancel{4x} + \cancel{4x} - 16$

$= x^2 - 16$

The *difference of squares* formula:

$(A+B)(A-B) = A^2 - B^2$

Example 5: Multiply

a.) $(r-7)(r+7)$

$= r^2 - 49$

b.) $(3xy+2z^2)(3xy-2z^2)$

$= (3xy)^2 - (2z^2)^2$

$= 9x^2y^2 - 4z^4$

c.) $\left(\frac{2}{3}n - m^3\right)\left(\frac{2}{3}n + m^3\right)$

$= \left(\frac{2}{3}n\right)^2 - (m^3)^2$

$= \frac{4}{9}n^2 - m^6$

d.) $(3x+5y)(-3x+5y)$

$= -9x^2 + \cancel{15xy} - \cancel{15xy} + 25y^2$

$= -9x^2 + 25y^2$

$(5y+3x)(5y-3x)$

$= 25y^2 - 9x^2$

Example 6: Multiply

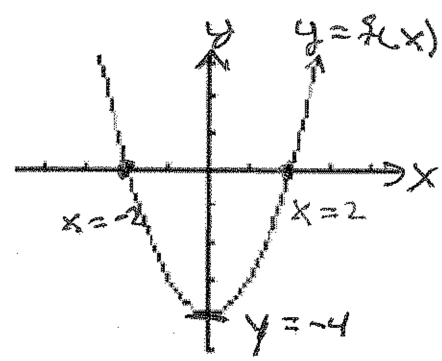
a.) $(2t-3)^2 - (t+2)(t-2)$

$= (2t)^2 - 2(2t)(3) + 9 - (t^2 - 4)$

$= 4t^2 - 12t + 9 - t^2 + 4$

$= 3t^2 - 12t + 13$

Connection with functions: The given graph shows $f(x) = (x-2)(x+2) = x^2 - 4$. Do you see any connections between the symbolic representation and the graph?



Example 7: Suppose $f(x) = x^2 - 3x + 2$. Find the following:

a.) $f(a) = a^2 - 3a + 2$

b.) $f(a)+3 = a^2 - 3a + 2 + 3$

$= a^2 - 3a + 5$

c.) $f(a+3) = (a+3)^2 - 3(a+3) + 2$

d.) $f(a+h)$

$= (a+h)^2 - 3(a+h) + 2$

$= a^2 + 2ah + h^2 - 3a - 3h + 2$

e.) $f(a+h) - f(a) = a^2 + 2ah + h^2 - 3a - 3h + 2 - (a^2 - 3a + 2)$

$= \cancel{a^2} + 2ah + h^2 - \cancel{3a} - 3h + \cancel{2} - \cancel{a^2} + \cancel{3a} - \cancel{2}$

$= 2ah + h^2 - 3h$