Functions (2.1) Math 98

Functions are used to show a relationship between two quantities. We will represent functions with words, tables, symbols, and graphs. Not all relationships can be considered functions however.

Function – a relationship between two sets of quantities, where

each input has exactly one output.

The Input – most common: x

set the of all inputs: domain

The Output – most common: y

set of all outputs: Range

Function Notation – $y = f(x)$

y : output

x : input

f : name of fct

note: fct = function.

A picture:

Wages Example –

min. wage @ hotels is $15/hr.

8 hr $\xrightarrow{w}$ $15 \cdot 8 = \$120$

32 hr $\xrightarrow{w}$ $15 \cdot 32 = \$480$

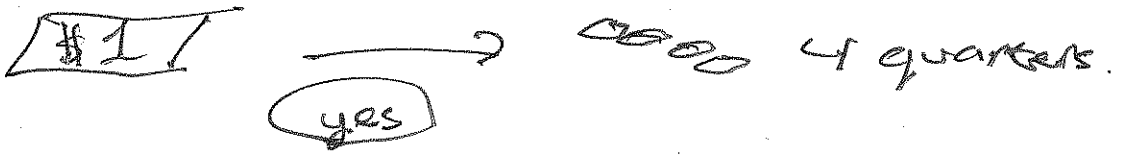
$t \xrightarrow{w} 15t$

$w(t) = 15 \cdot t$.

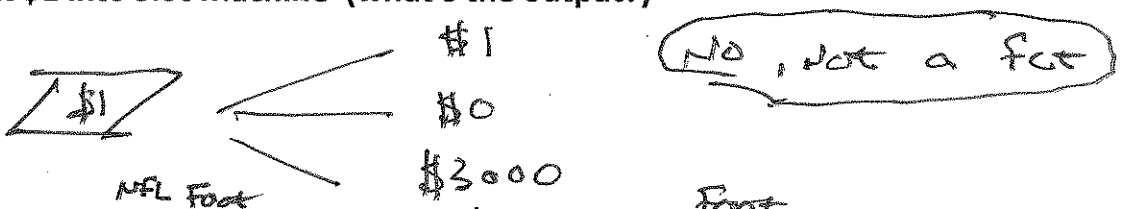
1. **Is it a Function?**

   a) **Input $1 to Quarter (25¢) Change Machine (what's the output?)**

   vs.



   Input $1 into Slot Machine (what's the output?)



   b)

Input – ~~Baseball~~ NFL Foot Team	Output – ~~Baseball~~ Foot City
Bengals	Cincinnati
Seahawks	Seattle
Broncos	Denver
Giants	New York
Jets	New York

Yes-a fct

   c)

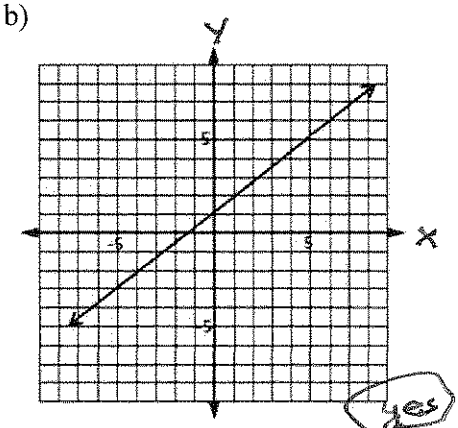
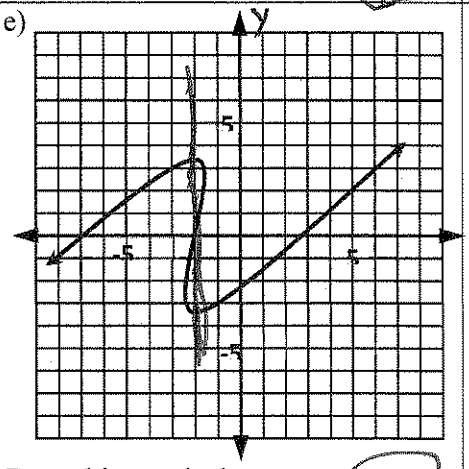
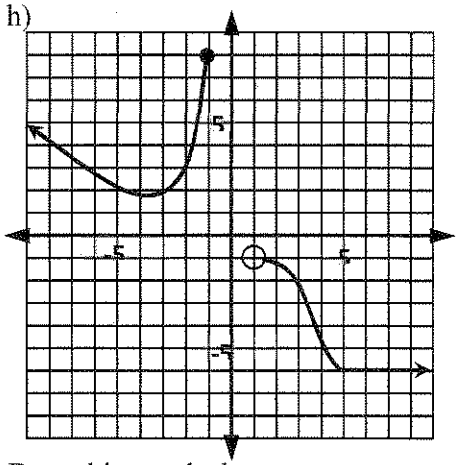
Input – ~~Baseball~~ Foot City	Output – ~~Baseball~~ fct Team
Seattle	Seahawks
~~Broncos~~ Denver	Broncos.
New York	Jets
	Giants.

No, not a fct

We need to be able to:

- Decide if a relationship is a function
- Evaluate a function
- Determine what numbers or values are allowed as inputs (Domain)
- Determine what numbers or values can result as outputs (Range)

2. For each of the examples in the table below, state whether the table, graph, or words do or do not describe a functional relationship. If it does not, provide an explanation or circle the features you used to make your decision.

Tables	Graphs	Words
a) | t | C | |---|---| | 1 | 1 | | 2 | 4 | | 3 | 9 | | 4 | 16 | | 5 | 25 | Is C a function of t? output / inputs yes	b)  yes	c) Is the amount you pay your babysitter for an evening out a function of the time you are gone if she charges \$6 an hour? $b(t) = 6t$ yes
d) | t | C | |---|---| | 15 | 100 | | 30 | 200 | | 45 | 100 | | 0 | 150 | | 15 | 100 | Is C a function of t? output / input yes	e)  Does this graph show y as a function of x? No vertical line test	f) Is the amount you are charged in sales tax a function of the cost of a taxable item purchased in Seattle? yes assuming comparable goods
g) | y | x | |---|---| | −3 | 5 | | −2 | 6 | | 0 | 7 | | −3 | 8 | | 1 | 9 | Is x a function of y? output / inputs No Is y a function of x? outputs / inputs yes	h)  Does this graph show y as a function of x? yes	i) Is the amount you are charged in federal income taxes a function of the amount you earn? No

Vertical Line Test – The graph of $y=f(x)$ is a function if all vertical lines intersect the graph at most once.

Evaluating Functions - The first step is being able to read and interpret the symbols in the name of a function.

$y = f(x)$ f for name

↖ "f of x"

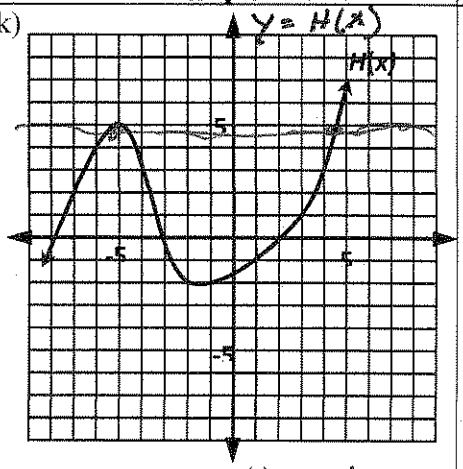
"f evaluated at x"

3. In the following examples, use the tables, graphs, and symbols to evaluate the given functions.

Tables	Graphs	Symbols
j)	k)	l) If $f(r)=2+7r-r^2$,

n	$p(n)$
6	21
−1	6 ←
2	13.5
0	9 ←
−4	−1

- Compute $p(0)$ = 9 (↑ N)
- Find $p(-1)$ = 6
- Find n such that $p(n)=13.5$ (↑ p-value) when $n=2$



- Compute $H(1)$ = -1
- What is $H(-7)$ = 2
- For what values of x is $H(x)=5$ (↑ y-val) $x=-5$ OR $x=4.5$

- Compute $f(3)$

$f(3) = 2+7(3)-3^2$

$= 14$

- Evaluate $f(r)$ at $r=8$

$f(8) = 2+7(8)-8^2$

$= -6$

- Find $f(-2)$

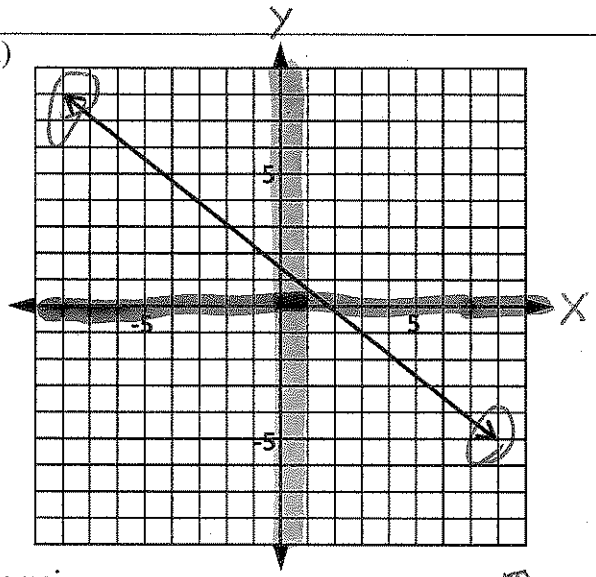
$f(-2) = 2+7(-2)-(-2)^2$

$= -16$

Identifying domains and ranges of various functions

4. Determine the domain and range of each of the following functions. Write your answers using interval notation.

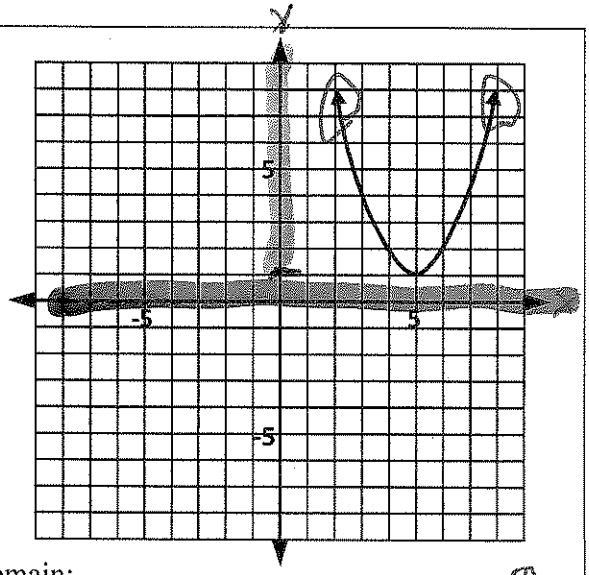
m)



Domain: $(-\infty, \infty)$ $x \in \mathbb{R}$

Range: $(-\infty, \infty)$ $y \in \mathbb{R}$

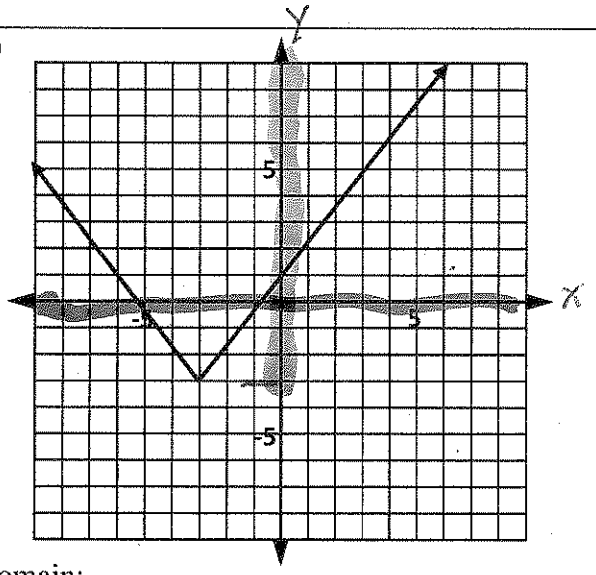
n)



Domain: $(-\infty, \infty)$ $x \in \mathbb{R}$

Range: $[1, \infty)$ $1 \leq y$ $y \geq 1$

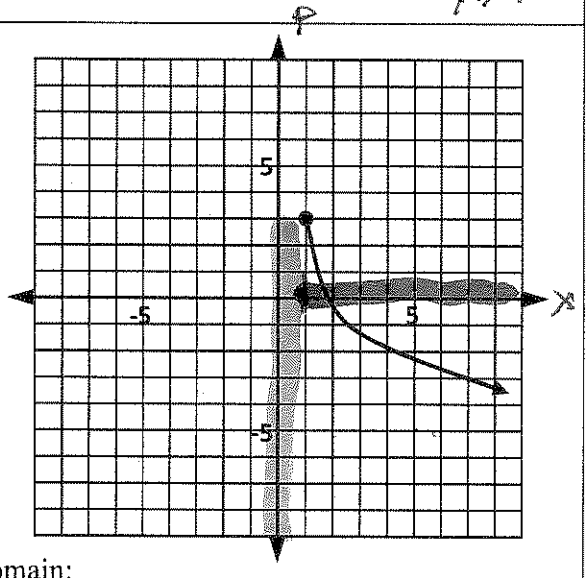
o)



Domain: $(-\infty, \infty)$ $x \in \mathbb{R}$

Range: $[-3, \infty)$ $-3 \leq y$ $y \geq -3$

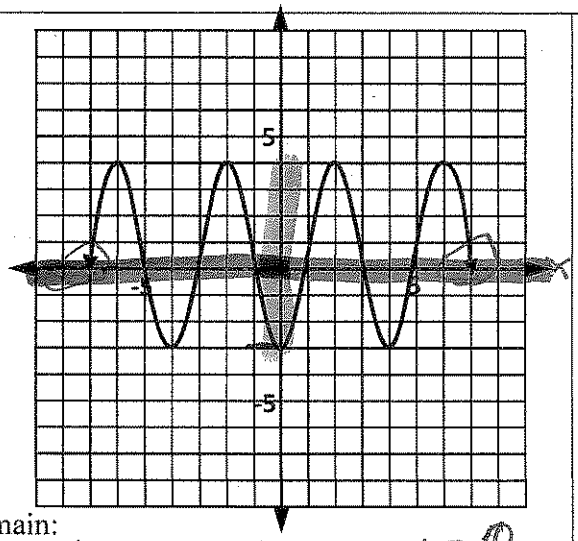
p)



Domain: $[1, \infty)$ $1 \leq x$ $x \geq 1$

Range: $(-\infty, 3]$ $y \leq 3$ $3 \geq y$

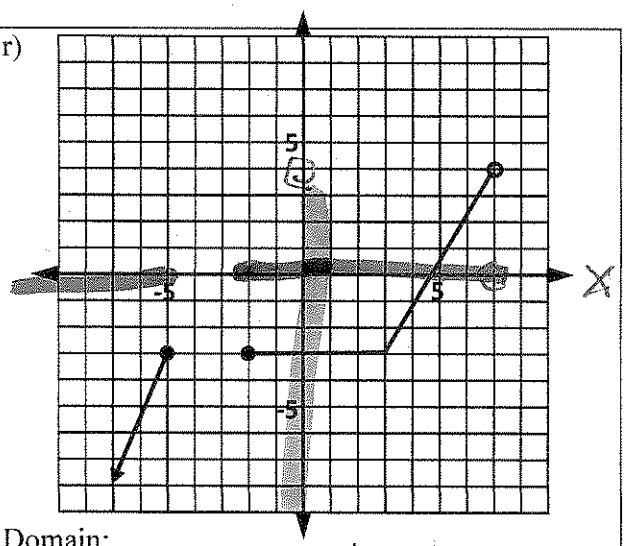
q)



Domain: $(-\infty, \infty)$ $x \in \mathbb{R}$

Range: $[-3, 4]$ $-3 \le y \le 4$

r)



Domain: $(-\infty, -5] \cup [-2, 7)$ $x \le -5$ or $-2 \le x < 7$

Range: $(-\infty, 4)$ $y < 4$

5. Determine the domain and range of functions given in tables, graphs, and symbols to evaluate the given functions.

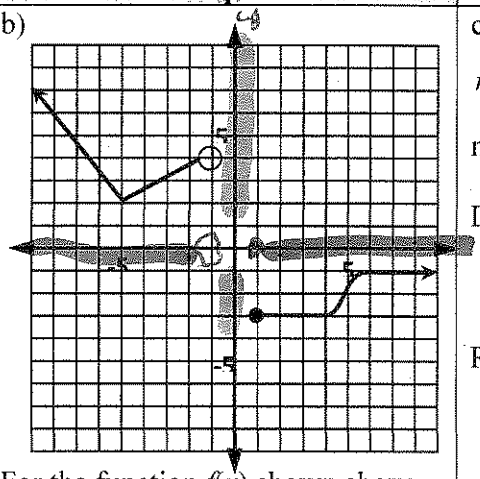
Tables	Graphs	Symbols
a)	b)	c) For the function $r(x) = \dfrac{8}{x(x-3)}$, use interval notation to describe its

h	W
37	592
42	672
37	592
35	560
48	768

For the function $W(h)$, use set notation to list its

Domain: $\{37, 42, 35, 48\}$

Range: $\{592, 672, 560, 768\}$



For the function $f(x)$ shown above, use interval notation to describe its

Domain: $(-\infty, -1) \cup [1, \infty)$ $x < -1$ or $1 \le x$

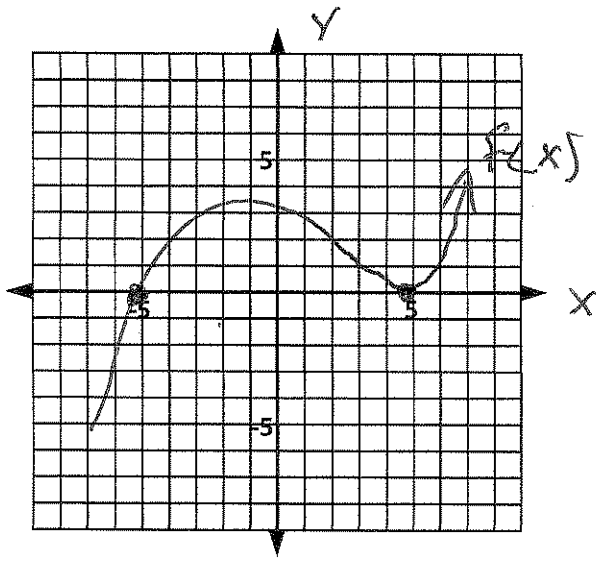
Range: $[-3, -1] \cup [2, \infty)$ $-3 \le y \le -1$ or $2 \le y$

Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$ $x \in \mathbb{R}$ except 0 and 3 and $x \neq 0, 3$

Range: $(-\infty, -3.5] \cup (0, \infty)$ $y \le -\frac{32}{9}$ or $0 < y$

A **ZERO** of a function means that:

- where the graph crosses or touches the x-axis.
- $y = 0$
- $f(x) = 0$



6. Find the zeros of functions.

a) $f(x) = 3x - 7$

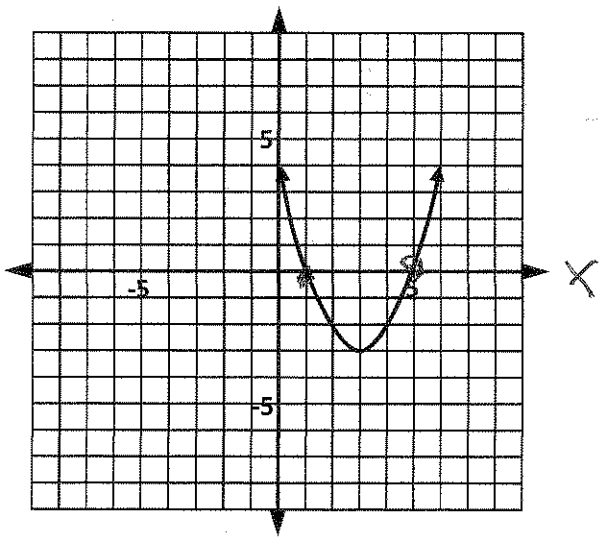
Solve $f(x) = 0$

$\Rightarrow 3x - 7 = 0$

$\Rightarrow 3x = 7$

$x = \frac{7}{3}$

b)



$x = 1$ or $x = 5$