Test 3
Dusty Wilson
Math 151

No work = no credit

No Symbolic Calculators

Name: Key

> *To divide a cube into two other cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it.*
>
> Pierre de Fermat (1601 - 1665)
> French mathematician

Warm-ups (1 pt each): $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2} = -x^{-2}$ $\quad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$ $\quad \frac{d}{dx}\frac{1}{\sqrt{x}} = \frac{1}{-2x^{3/2}} = -\frac{1}{2}x^{-3/2}$

1.) (1 pt) According to Fermat (see above), how long do you think his admirable proof was?

He thought it would be short... he was wrong.

2.) (10 pts) Consider $f(x) = 3x^4 - 7\sqrt{x} + 3\sin(x)$

a.) (8 pts) Find the general antiderivative $F(x)$

$$F(x) = \frac{3}{5}x^5 - \frac{7x^{3/2}}{3/2} - 3\cos x + C$$

$$= \frac{3}{5}x^5 - \frac{14}{3}x^{3/2} - 3\cos x + C$$

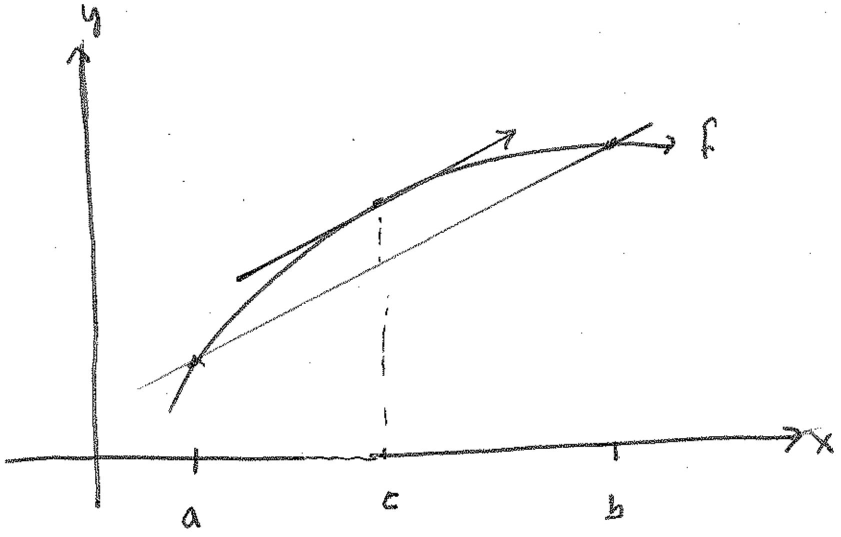
b.) (2 pts) If $F(0) = 31$ find the specific antiderivative $F(x)$

$$F(0) = -3 + C = 31 \Rightarrow C = 34$$

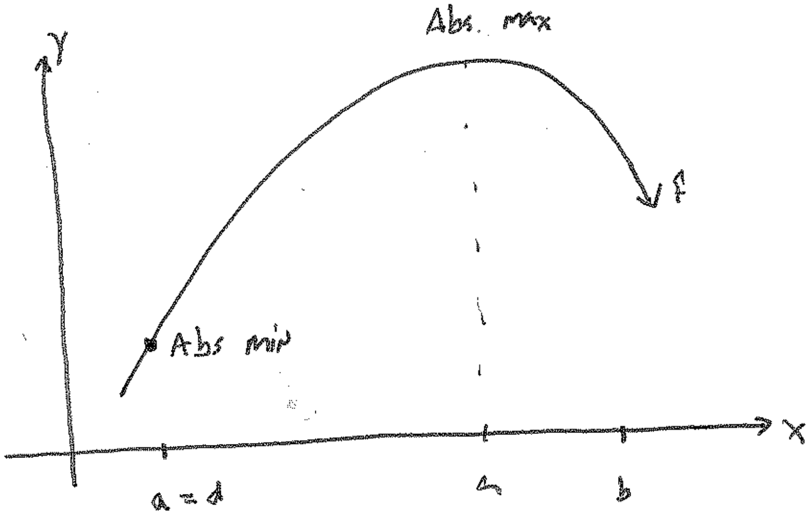
$$F(x) = \frac{3}{5}x^5 - \frac{14}{3}x^{3/2} - 3\cos x + 34$$

3.) (10 pts) State either the Mean Value Theorem (MVT) or Extreme Value Theorem (EVT). Sketch a picture that explains the theorem.

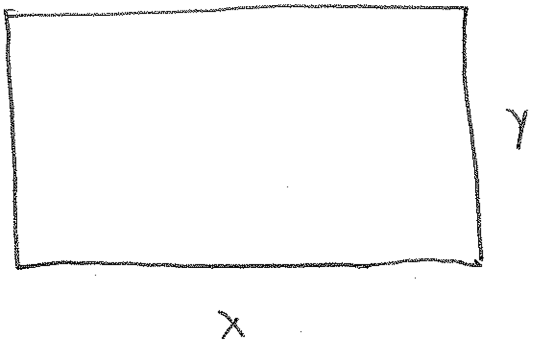
MVT: If f is continuous on $[a,b]$ and differentiable on (a,b) then there is $c \in (a,b)$ s.t. $f'(c) = \frac{f(b) - f(a)}{b - a}$



EVT: If f is continuous on $[a,b]$ then f attains an abs. max $f(c)$ and abs. min $f(d)$ at some $c,d \in [a,b]$



4.) (10 pts) The base of a rectangle is increasing by 4 cm/s while its height is decreasing by 3 cm/s. At what rate is the area changing when the base is 20 cm and the height is 12 cm?



1pt.

Area: $A = xy$

we know $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = -3$ ↓

we want $\frac{dA}{dt}$

Let's do implicit differentiation

$$\frac{d}{dt} A = \frac{d}{dt}(xy)$$

$$\Rightarrow \frac{dA}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x$$

(-3 if no product rule).

$$\uparrow \quad \uparrow \qquad \uparrow \quad \uparrow$$

$$4 \cdot 12 + -3 \cdot 20$$

$$= -12$$

The area is decreasing by 12 cm^2/s.

5.) (10 pts) Evaluate $\lim_{x->0} \frac{1}{x}\ln\left(\frac{7x+8}{4x+8}\right)$ $= \lim_{x\to 0} \frac{\ln\left(\frac{7x+8}{4x+8}\right)}{x}$

$$= \lim_{x\to 0} \frac{\ln(7x+8) - \ln(4x+8)}{x}$$

$$\overset{\textcircled{H}}{=} \lim_{x\to 0} \frac{\frac{7}{7x+8} - \frac{4}{4x+8}}{1}$$

$$= \frac{7}{8} - \frac{4}{8}$$

$$= \frac{3}{8}$$

6.) (10 pts) Evaluate $\lim_{x->0} (\cos x)^{1/x^2}$

Let $y = \lim_{x\to 0} (\cos x)^{1/x^2}$

$$\Rightarrow \ln y = \lim_{x\to 0} \ln\left[(\cos x)^{1/x^2}\right]$$

$$= \lim_{x\to 0} \frac{\ln(\cos x)}{x^2}$$

$$\overset{\textcircled{H}}{=} \lim_{x\to 0} \frac{-\tan x}{2x}$$

$$\overset{\textcircled{H}}{=} \lim_{x\to 0} \frac{-\sec^2 x}{2}$$

$$= -\frac{1}{2}$$

and $y = e^{-1/2}$

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We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1623 - 1662)
French mathematician

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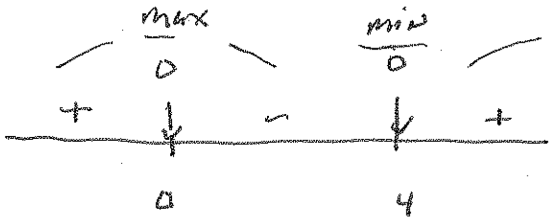
No Calculators

1.) (10 pts) Find and clearly label all local and absolute extremes of $f\left(x\right)=x^3-6x^2+4$ on $\left[-3,5\right]$

$f'(x) = 3x^2 - 12x$

$= 3x(x-4)$



$f(-3) = -27 - 6(9) + 4$

$= -27 - 54 + 4$

$= -77$

$f(4) = 64 - 6(16) + 4$

$= 64 - 96 + 4$

$= -28$

X	Y	description
-3	-77	abs. min.
0	4	abs. max
4	-28	local min
5	-21	

$f(5) = 125 - 6(25) + 4$

$= 125 - 150 + 4$

$= -21$

$\frac{8}{10}$ if endpts not checked.

2.) (10 pts) Consider the following function and its derivatives:

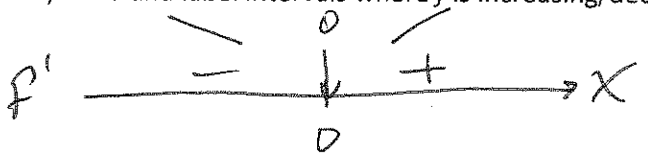
$$f(x)=2+\ln\left(x^2+1\right)$$

$$f'(x)=\frac{2x}{x^2+1}$$

2 pts if not factored.

$$f''(x)=\frac{2\left(1-x^2\right)}{\left(x^2+1\right)^2} = \frac{2(1-x)(1+x)}{(x^2+1)^2}$$

a.) Find and label intervals where *f* is increasing/decreasing.



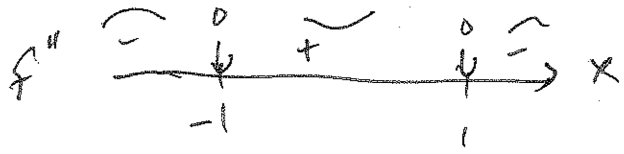
increasing $(0,\infty)$

decreasing $(-\infty,0)$

b.) Find and label all local extremes (label as local min/max or absolute min/max)

local min @ $(0,2)$

c.) Find and label intervals where *f* is concave up/down.



concave up $(-1,1)$

concave down

$(-\infty,-1)\cup(1,\infty)$

d.) Find a point of inflection.

POI $(-1, 2+\ln 2)$

$(1, 2+\ln 2)$

$\overline{10}$

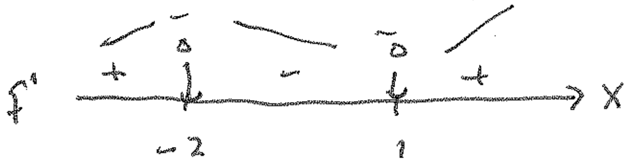
2.) (10 pts) Consider $f(x) = 2x^3 + 3x^2 - 12x$.

a.) Find and label intervals where *f* is increasing/decreasing.

$f'(x) = 6x^2 + 6x - 12$

$= 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$



increasing

$(-\infty, -2) \cup (1, \infty)$

decreasing

$(-2, 1)$

b.) Find and label all local extremes.

$f(-2) = 2(-8) + 3(4) + 24 = 20$

$f(1) = 2 + 3 - 12 = -7$

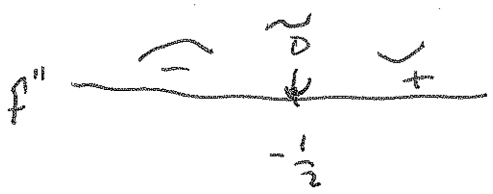
local max @ $(-2, 20)$

local min @ $(1, -7)$

c.) Find and label intervals where *f* is concave up/~~down~~.

$f''(x) = 12x + 6$

$= 6(2x+1)$



concave up on $(-\frac{1}{2}, \infty)$

d.) Find all points of inflection.

$f(-\frac{1}{2}) = 2(-\frac{1}{8}) + 3(\frac{1}{4}) + 6$

$= -\frac{1}{4} + \frac{3}{4} + 6$

$= \frac{13}{2}$

Point of inflection $(-\frac{1}{2}, \frac{13}{2})$