Test 2
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Math 151

Name: key.

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1623 - 1662)
French mathematician

No work = no credit

No Symbolic Calculators

Warm-ups (1 pt each): $\frac{d}{dx}\pi = 0$ $\frac{d}{dx}x^{\pi} = \pi x^{\pi-1}$ $\frac{d}{dx}\pi^{x} = \pi^{x}\ln(\pi)$

1.) (1 pt) According to Pascal (see above), what kind of logic do we find most compelling?

That logic which we find ourself.

2.) (12 pts) Differentiate $h(x) = 9x^8 + 7\sqrt[6]{x} + \frac{5}{x^4} + 3\log_2(x) + 9^x + 8\tan^{-1}(x)$. Express your answer as an equation.

$$h'(x) = 72x^7 + \frac{7}{6}x^{-5/6} - 20x^{-5} \quad \frac{3}{\ln 2 \cdot x} + 9^x \ln 9 + \frac{8}{1+x^2}$$

3.) (10 pts) Find $\frac{d}{dx}2x^3\sin(x) = 6x^2\sin x + 2x^3\cos x$

4.) (10 pts) If $g(x) = \frac{3\sqrt{x}}{5x^2-2x+7} = 3x^{1/2}(5x^2-2x+7)^{-1}$

$$g'(x) = \frac{\frac{3}{2\sqrt{x}}(5x^2-2x+7) - 3\sqrt{x}(10x-2)}{(5x^2-2x+7)^2}$$

$$= \frac{3}{2}x^{-1/2}(5x^2-2x+7) - (5x^2-2x+7)^{-2}(10x-2)\cdot 3x^{1/2}$$

5.) (10 pts) If $f(x) = 2\tan^{-1}\left(e^{x^5+3}\right)$ find $\frac{df}{dx}$.

$$\frac{df}{dx} = \frac{2}{1+\left(e^{x^5+3}\right)^2} \cdot e^{x^5+3} \cdot 5x^4$$

6.) (10 pts) Find the <u>equation</u> of the second derivative of $y = \ln\left(\cos\left(x^7+1\right)\right)$

$$\Rightarrow \quad y' = \frac{1}{\cos(x^7+1)} \cdot -\sin(x^7+1) \cdot 7x^6$$

$$= -7x^6 \tan(x^7+1)$$

$$\Rightarrow \quad y'' = -42x^5 \tan(x^7+1) + -7x^6 \sec^2(x^7+1) \cdot 7x^6$$

7.) (10 pts) Differentiate $y = \left(\overset{\tan}{\cancel{\cos}}(x)\right)^{4x}$. Express y' in terms of x.

$$\Rightarrow \quad \ln y = 4x \ln(\tan x)$$

$$\Rightarrow \quad \frac{d}{dx} \ln y = \frac{d}{dx} 4x \ln(\tan x)$$

$$\Rightarrow \quad \frac{y'}{y} = 4\ln(\tan x) + 4x \cdot \frac{1}{\tan x} \cdot \sec^2 x$$

$$\Rightarrow \quad y' = (\tan x)^{4x} \left(4\ln(\tan x) + \frac{4x\sec^2 x}{\tan x}\right)$$

8.) (10 pts) If $q = \dfrac{(3z^2-1)^4}{\sin(5z)\sqrt{6z+1}}$, find $\dfrac{dq}{dz}$. Give your answer as an <u>equation</u>.

$$\Rightarrow \ln q = 4\ln(3z^2-1) - \ln(\sin 5z) - \frac{1}{2}\ln(6z+1)$$

$$\Rightarrow \frac{q'}{q} = \frac{4\cdot 6z}{3z^2-1} - \frac{5\cos 5z}{\sin 5z} - \frac{3}{6z+1}$$

$$\Rightarrow \frac{dq}{dz} = \frac{(3z^2-1)^4}{\sin(5z)\sqrt{6z+1}}\left(\frac{24z}{3z^2-1} - 5\cot(5z) - \frac{3}{6z+1}\right)$$

9.) (10 pts) Find the equation of the ~~normal~~ tangent line to $x^2+2xy-y^2+x=2$ at the point $(1,2)$.

$$\Rightarrow 2x+2y+2xy'-2yy'+1=0$$

$$\Rightarrow -2xy'+2yy' = +1+2x+2y$$

$$\Rightarrow y' = \frac{1+2x+2y}{2y-2x}\Bigg|_{(1,2)} \qquad \frac{7}{2}$$

$$y-2 = \frac{7}{2}(x-1)$$

10.) (10 pts) Find and evaluate the following given the table below. Circle your answers.

a.) $\frac{d}{dx}\left[f(x)g(x)\right]$ when $x=2$

x	1	2	3	4	5
$f(x)$	5	1	3	4	2
$f'(x)$	5	2	4	3	1
$g(x)$	4	1	2 (3)	2 (3)	5
$g'(x)$	1	5	4	2	3

$= f'(2)g(2) + g'(2)f(2)$

$= 2(1) + 5(1)$

$= 7$

b.) $\frac{d}{dx}f\left[g\left(x^2\right)\right]$ when $x=2$

$= f'(g(4)) \cdot g'(4) \cdot 2(2)$

$= f'(3) \cdot 2 \cdot 4$

$= 4 \cdot 2 \cdot 4$

$= 32$

11.) (10 pts) Differentiate $y=3\left(\frac{4x+1}{\ln(6x)}\right)^{-5}$ with respect to *x*.

$$= -15\left(\frac{4x+1}{\ln 6x}\right)^{-6} \cdot \frac{4\ln(6x) - (4x+1)\cdot\frac{6}{6x}}{[\ln(6x)]^2}$$