**Test 1**
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Math 151

Name: KEY

*It is rare to find learned men who are clean, do not stink and have a sense of humour.*

Gottleb Leibnez (1646 - 1716)
German mathematician

**No work = no credit**

**No Symbolic Calculators**

Warm-ups (1 pt each): $-2^2 =$ -4 $\quad -1^0 =$ -1 $\quad 2+2 =$ 4

1.) (1 pt) The quote (above) was said about Leibniz. What were two of his positive qualities?

Leibniz was clean, didn't stink, & had a good sense of humor

2.) (5 pts) The table gives the values of *f* near 2, but not equal to 2. Use the table to estimate $\lim_{x\to 2} f(x)$.

| $x$ | 1 | 1.9 | 1.99 | 2 | 2.01 | 2.1 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.8325 | 3.0251 | 2.9998 | 3 | 3.0001 | 2.9769 | 2.4984 |

$\lim_{x\to 2} f(x) = 3$

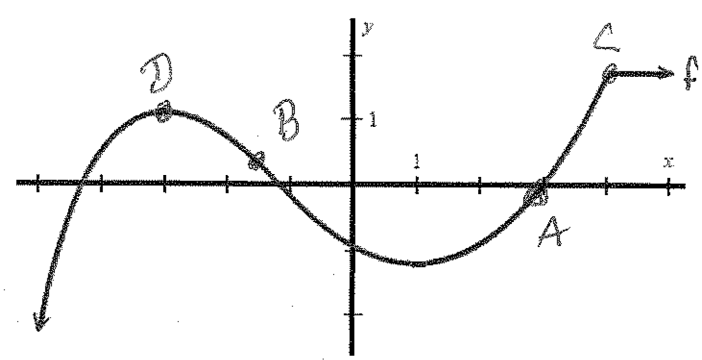
3.) (8 pts) Consider the graph of *f* (right).

a.) Find and label a point A where $f'$ is positive.

b.) Find and label a point B where $f'$ is negative.

c.) Find and label a point C where $f'$ is undefined.

d.) Find and label a point D where $f'$ is zero and is negative immediately to the left of D and positive to the right of D.



4.) (10 pts) Evaluate $\lim_{x\to\infty} \frac{2x+1}{\sqrt{9x^2+5x}-3}$ $= \lim_{x\to\infty} \frac{\frac{2x+1}{x}}{\frac{\sqrt{9x^2+5x}-3}{x}}$

$= \lim_{x\to\infty} \frac{2+\frac{1}{x}}{\sqrt{9+\frac{5}{x}}-\frac{3}{x}}$

$= \frac{2}{\sqrt{9}}$

$= \frac{2}{3}$

5.) (10 pts) Consider the function $f(x)=3x^2+4x+5$.

a.) Use the <u>definition of the derivative</u> to find the derivative of $f$. Hint: You may check using the techniques of chapter 3.

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{3(x+h)^2+4(x+h)+5-(3x^2+4x+5)}{h}$

$= \lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2+\cancel{4x}+4h+\cancel{5}-\cancel{3x^2}-\cancel{4x}-\cancel{5}}{h}$

$= \lim_{h\to 0}(6x+3h+4)$

$= 6x+4$

b.) Find the equation of the tangent line to $f$ when $x$ = -1.

pt: $(-1, 4)$

slope: $f'(-1) = -2$

Tangent line: $y-4=-2(x-(-1))$

$\Rightarrow y=-2x+2$

6.) (5 pts) Using the definition of continuity, explain why the function $f(x)=\begin{cases}\cos(x), x<0\\ 2 \quad, x=0\\ 1-x^2 \ , x>0\end{cases}$ is discontinuous at $x=0$. Show work to support your explanation (one sided limits).

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \cos x = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (1-x^2) = 1$

so $\lim_{x\to 0} f(x) = 1$.

but $f(0)=2$

$\Rightarrow$ so f is discont. since $\lim_{x\to 0} f(x) \neq f(0)$.

7.) (5 pts) Use the precise definition of the limit to prove $\lim_{x\to 4}(2x+3)=11$

claim: $\lim_{x\to 4}(2x+3)=11$

proof.

Let $\varepsilon>0$ be given.

choose $\delta=\frac{\varepsilon}{2}$.

If $0<|x-4|<\delta$

$\Rightarrow |x-4|<\frac{\varepsilon}{2}$

$\Rightarrow 2|x-4|<\varepsilon$

$\Rightarrow |2x-8|<\varepsilon$

$\Rightarrow |(2x+3)-11|<\varepsilon$

Hence $\lim_{x\to 4}(2x+3)=11$.

8.) (10 pts) Evaluate $\lim_{x\to 2} \frac{\sqrt{9x-2}-4}{x-2} \cdot \frac{\sqrt{9x-2}+4}{\sqrt{9x-2}+4}$

$$= \lim_{x\to 2} \frac{9x-2-16}{(x-2)(\sqrt{9x-2}+4)} \longleftarrow 9x-18 = 9(x-2)$$

$$= \lim_{x\to 2} \frac{9}{\sqrt{9x-2}+4}$$

$$= \frac{9}{8}$$

9.) (5 pts) State either the <u>definition of the derivative</u> or the <u>intermediate value theorem</u> (your choice)

10.) (5 pts) If $2x \le g(x) \le x^4 - x^2 + 2$ for all *x*, evaluate $\lim_{x \to 1} g(x)$

$\lim_{x \to 1} 2x = 2 = \lim_{x \to 1} (x^4 - x^2 + 2)$

Hence $\lim_{x \to 1} g(x) = 1$ by the squeeze thm

11.) (6 pts) Suppose the height of a falling object after *t* seconds is given by the function $s(t) = 100 - 16t^2$ where the position is given in feet above the ground.

a.) Find and interpret $s(2)$

$s(2) = 36.$
the object is 36 ft up after 2 seconds

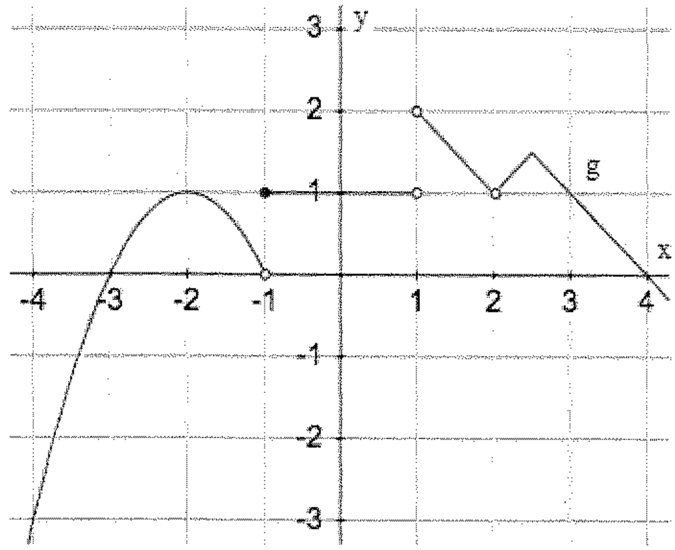
b.) Use the <u>definition of the derivative</u> to find $s'(2)$

$$s'(2) = \lim_{h \to 0} \frac{s(2+h) - s(2)}{h}$$
$$= \lim_{h \to 0} \frac{100 - 16(2+h)^2 - 36}{h}$$
$$= \lim_{h \to 0} \frac{100 - 64 - 64h - 16h^2 - 36}{h}$$
$$= \lim_{h \to 0} \frac{-64h - 16h^2}{h}$$
$$= \lim_{h \to 0} (-64 - 16h)$$
$$= -64$$

c.) Interpret $s'(2)$ including units

After 2 seconds the ball is falling @ 64 ft/s.

12.) (16 pts) Use the graph of $g$ to answer the questions below.




a.) (2 pts) $\lim_{x \to 2} g(x) =$ 1

b.) (2 pts) Find $\lim_{x \to -1^+} g(x) =$ 1

c.) (2 pts) Find $\lim_{x \to -1} g(x) =$ DNE

d.) (2 pts) Find $g'(3) =$ -1

$\frac{1}{2}$ for "1"

e.) (2 pts) Where is $g'$ zero?

$x = -2$ or $-1 \leq x < 1$ $\qquad$ $\frac{1}{2}$ if one answer only.

f.) (2 pts) For what $a$ does $\lim_{x \to a} g(x)$ exist, but $g$ does not exist?

$a = 2$

g.) (2 pts) For what $b$ does $\lim_{x \to b} g(x)$ fail to exist, but $g$ does exist?

$b = -1$

h.) (2 pts) Give an $x$ value such that $g(x) > 0$ and $g'(x) < 0$ (Hint: There is more than one answer, but you only need to give one.)

$x = -1.5$