Test 2
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Math 151

Name: Key

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1623 - 1662)
French mathematician

No work = no credit

No Symbolic Calculators

Warm-ups (1 pt each): $\frac{d}{dx}x = $ 1 $\qquad \frac{d}{dy}x = \frac{dx}{dy}$ or 0 $\qquad \frac{d}{dz}1 = $ 0

1.) (1 pt) According to Pascal (see above), what kind of logic do we find most compelling?

We are most convinced by our own "revelations."

2.) (12 pts) Differentiate $h(x)=3x^8+4\sqrt[5]{x}+\frac{7}{x^6}-\log_9(x)-2^x+8\sin^{-1}(x)$. Express your answer as an <u>equation</u>.

$$h'(x) = 24x^7 + \frac{4}{5}x^{-4/5} - 42x^{-7} - \frac{1}{x\ln 9} - 2^x \ln 2 + \frac{8}{\sqrt{1-x^2}}$$

3.) (10 pts) Find the <u>equation</u> of the second derivative of $f(x)=\sin(3x^5)$.

$$f'(x) = 15x^4 \cos(3x^5)$$

$$f''(x) = 60x^3 \cos(3x^5) - (15x^4)^2 \sin(3x^5)$$

26

91

5 pts.

4.) (10 pts) Find $\frac{d}{dx}\tan^{-1}\left(\sqrt{3x+2}\right)$

$$= \frac{1}{1+\left(\sqrt{3x+2}\right)^2} \cdot \frac{1}{2\sqrt{3x+2}} \cdot 3$$

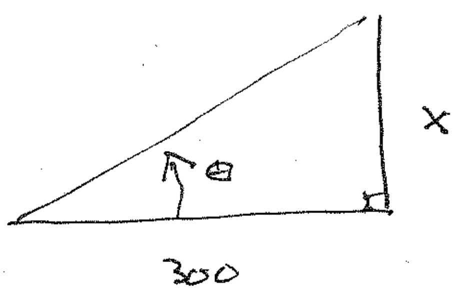
$$= \frac{1}{3x+3} \cdot \frac{3}{2\sqrt{3x+2}}$$

$$= \frac{1}{2(x+1)\sqrt{3x+2}}$$

5.) (10 pts) An observer stands 300 feet from the launch site of a hot-air balloon. The balloon is launched vertically and maintains a constant upward velocity of 20 ft/s. What is the rate of change of the angle of elevation of the balloon when it is 400 feet from the ground?

(Note: The angle of elevation is the angle between the observer's line of sight to the balloon and the ground.)

Your answer should be in sentence form, and should include the appropriate units.



The angle of elevation is growing at $\frac{3}{125}$ rad/s

0.024 rad/s

1.375 deg/s

$\tan\theta = \frac{x}{300}$

5 pts

$$\Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{1}{300}\frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{\cos^2\theta}{300}\frac{dx}{dt}$$

$\frac{dx}{dt} = 20$

$\cos\theta = \frac{3}{5}$

$$\frac{\left(\frac{3}{5}\right)^2 \cdot 2\cancel{0}}{30\cancel{0}} = \frac{\cancel{18}^{\,3}}{25 \cdot \cancel{30}_{\,5}} = \frac{3}{125}$$

16 ft/s

b for

10

20

6.) (10 pts) Use a linear approximation (or differentials) to estimate the number $3^{1.95}$

Let $f(x) = 3^{x}$ @ $(2, 9)$

$f'(x) = 3^x \ln 3 \Big|_{x=2} \quad 9\ln 3$

Tangent: $y - 9 = 9\ln 3 (x - 2)$

$\Rightarrow \quad y = 9\ln 3(x-2) + 9 \Big|_{x=1.95} \qquad 8.5056$

$3^{1.95} \simeq 8.5056$

7.) (10 pts) Differentiate $y = (\csc(x))^{42x}$. Express y' in terms of x.

$\ln y = 42x \ln(\csc x)$

$\Rightarrow \dfrac{y'}{y} = 42\ln(\csc x) + 42x \cdot \dfrac{1}{\cancel{\csc} x} \cdot -\cancel{\csc} x \cot x$

$\Rightarrow y' = (\csc x)^{42x} \left(42\ln(\csc x) - 42x \cot x\right)$

8.) (10 pts) If $q = \dfrac{\sqrt{\tan(z)}}{\ln(z)}$, find $\dfrac{dq}{dz}$. ~~Give your answer as an equation.~~ 2 pts.

$$\frac{dq}{dz} = \frac{\frac{1}{2\sqrt{\tan z}} \cdot \sec^2 z \ln z - \frac{1}{z} \cdot \sqrt{\tan z}}{[\ln z]^2}$$

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9.) (10 pts) Find the equation of the normal line to $x^4 - x^2y + y^4 = 1$ at the point $(-1,1)$.

$$\frac{d}{dx}(x^4 - x^2y + y^4) = \frac{d}{dx}(1)$$

$$\Rightarrow 4x^3 - 2xy - x^2y' + 4y^3y' = 0$$

$$\Rightarrow y' = \frac{2xy - 4x^3}{4y^3 - x^2}\Bigg|_{(x,y)=(-1,1)} \quad \frac{-2+4}{4-1} = \frac{2}{3}$$

slope of normal $-\frac{3}{2}$

$$y - 1 = -\frac{3}{2}(x+1)$$

$$y = -\frac{3}{2}x - \frac{1}{2}$$

8/10 for tangent

10.) (10 pts) Find and evaluate the following given the table below. Circle your answers.

a.) $\frac{d}{dx}\left[f(x) + 2g(x)\right]$ when $x = 3$

x	1	3	5	7	9
$f(x)$	3	1	9	7	5
$f'(x)$	7	9	5	1	3
$g(x)$	9	7	5	3	1
$g'(x)$	5	9	3	1	7

$$f'(x) + 2g'(x)$$

$$@\ x = 3:\ 9 + 2(9) = 27$$

b.) $\frac{d}{dx} f\left[g(x^2)\right]$ when $x = 3$

$$f'(g(x^2)) \cdot g'(x^2) \cdot 2x$$

$$@\ x = 3:\ f'(\underbrace{g(9)}_{1}) \cdot \underbrace{g'(9)}_{7} \cdot 6 = 294$$

$$\underbrace{f'(1)}_{7} \cdot 7 \cdot 6$$

70