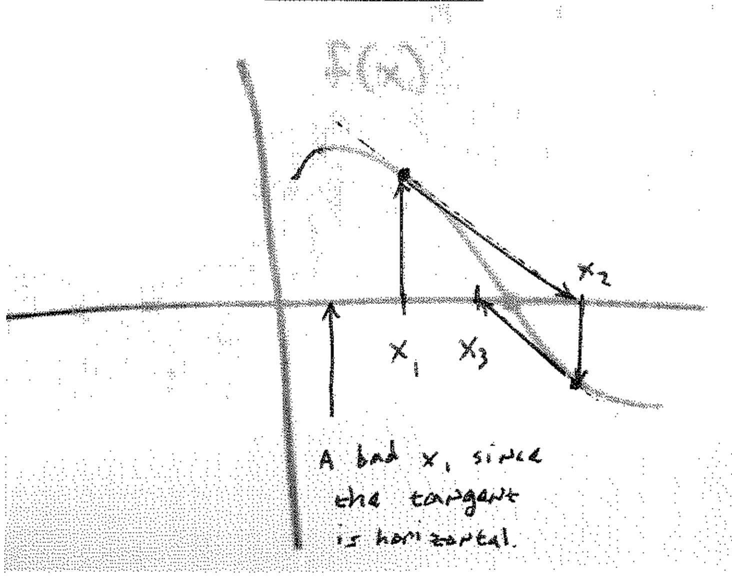
Math 151
Newton's Method

Video 1: "Newton's Method" with URL https://youtu.be/1uN8cBGVpfs

Concept: (start 0:00 and end 2:10) Newton's Method

Newton's Method



Start with a given guess: x_1

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

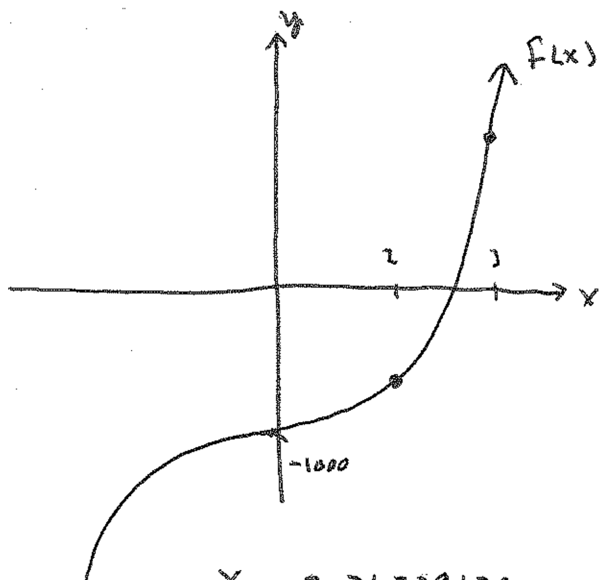
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$\vdots$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example 1: (start 2:10 and end 7:29) Find where $f(x) = x^7 - 1000$ intersects the x-axis; find the solution correct to 8 decimal places.



$$x_{n+1} = x_n - \frac{x_n^7 - 1000}{7x_n^6}$$

$$x_1 = 3$$

$$x_2 = 3 - \frac{3^7 - 1000}{7(3)^6}$$

$$\approx 2.76739173$$

$$x_3 = 2.76739173 - \frac{2.76739173^7 - 1000}{7(2.76739173)^6}$$

$$\approx 2.69008741$$

$$x_4 \approx 2.68275645$$

$$x_5 \approx 2.68269580$$

$$x_6 \approx 2.68269580$$

Video 2: "Newton's Method - More Examples Part 1 of 3" with URL https://youtu.be/xdLgTDIFwrc

Example 2: (start 0:00 and end 6:53) Compute two iterations of Newton's Method for the function $f(x) = x^2 - 8$ for the initial guess $x_1 = 3$.

$f(x) = x^2 - 8$

$\Rightarrow f'(x) = 2x$

$x_1 = 3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 3 - \frac{3^2 - 8}{2(3)}$$

$$= 3 - \frac{1}{6}$$

$$= \frac{17}{6}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= \frac{17}{6} - \frac{\left(\frac{17}{6}\right)^2 - 8}{2\left(\frac{17}{6}\right)}$$

$$\approx 2.833 - \frac{8.026 - 8}{5.667}$$

$$\approx 2.833 - 0.005$$

$$= 2.828$$

check: $f(2.828) = 2.828^2 - 8$

$\approx 7.998 - 8$

$= -0.002$

close to zero so it's working.

Video 3: "Newton's Method - More Examples Part 2 of 3" with URL https://youtu.be/wFubpuCNB_w

Example 3: (start 0:00 and end 5:13) Apply two iterations of Newton's Method to approximate the x-value of a point of intersection of the two functions $f(x)=x^2-4$ and $g(x)=2x-3$ for the initial guess $x_1=0$. That is, what is the value of x_3?

To find the intersection solve $f(x)=g(x)$

$\Rightarrow x^2-4=2x-3$

$\Rightarrow \underbrace{x^2-2x-1}_{h(x)}=0$

$h(x)=x^2-2x-1$

$\Rightarrow h'(x)=2x-2$

$x_1=0$

$$x_2=0-\frac{0^2-2(0)-1}{2(0)-2}$$

$$=-\frac{1}{2}$$

$$x_3=-\frac{1}{2}-\frac{\left(-\frac{1}{2}\right)^2-2\left(-\frac{1}{2}\right)-1}{2\left(-\frac{1}{2}\right)-2}$$

$$=-\frac{1}{2}-\frac{\frac{1}{4}+\cancel{1}-\cancel{1}}{-1-2}$$

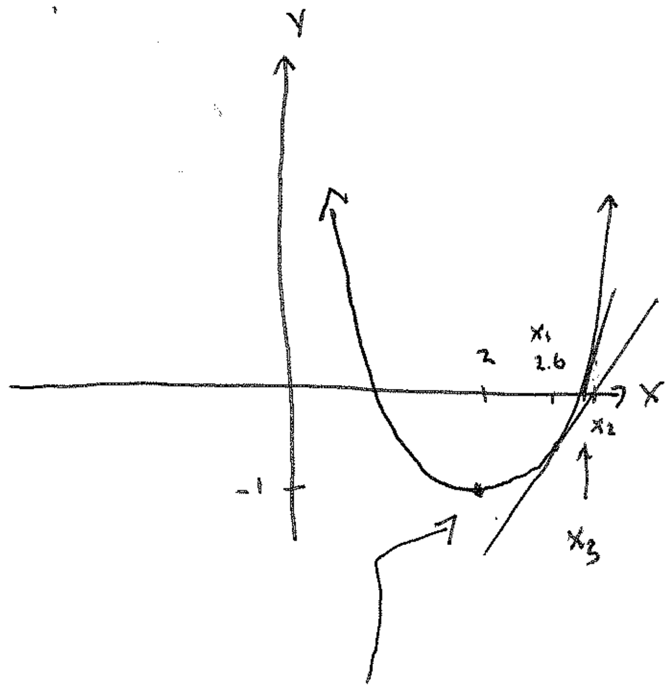
$$=-\frac{1}{2}+\frac{1/4}{3}$$

$$=-\frac{1}{2}+\frac{1}{12}$$

$$=-\frac{5}{12}$$

Video 4: "Newton's Method - *How it Can FAIL* - More Examples Part 3 of 3" at the URL https://youtu.be/9RjI_so9oSM

Example 4: (start 0:00 and end 3:47) Given the equation $f(x)=(x-2)^2-1$ and initial guess $x_1=2$, why would Newton's Method fail to approximate a solution?



Tangent @ $x=2$
is horizontal.
FAIL!