

2.6: Limits @ Infinity.

Ex1: Numerically explore $\lim_{x \rightarrow \infty} 3 + \frac{1}{x} \sin x$

Dfn: Let f be a fcn defined on some interval (a, ∞) . Then $\lim_{x \rightarrow \infty} f(x) = L$ means that values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

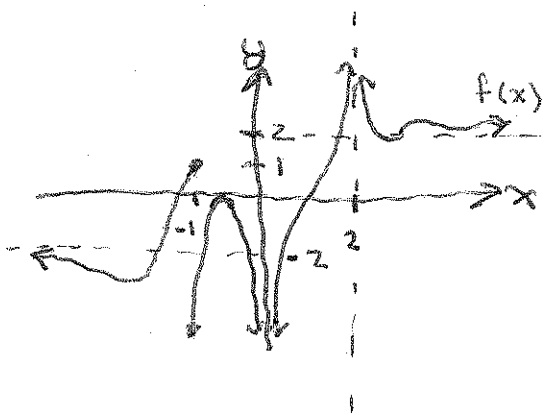
more precisely: $\forall \epsilon > 0 \exists N \in \mathbb{N}$ s.t.

if $x > N$ then $|f(x) - L| < \epsilon$

We call the line $y = L$ a horizontal asymptote of $y = f(x)$ if $\lim_{x \rightarrow -\infty} f(x) = L$ or $\lim_{x \rightarrow \infty} f(x) = L$.

Ex2: Find infinite limits, limits @ infinity,

and the eqn. of hor. asy.



$$\lim_{x \rightarrow 2} f(x) = \infty$$

$$\lim_{x \rightarrow -1} f(x) \text{ DNE since}$$

$$\lim_{x \rightarrow -1^-} f(x) = 1$$

$$\lim_{x \rightarrow -1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = 2 \text{ (H.A.)}$$

$$\lim_{x \rightarrow -\infty} f(x) = -2 \text{ (H.A.)}$$

Important examples to know

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x}$$

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^2}$$

$$\lim_{x \rightarrow \pm\infty} \arctan(x)$$

Thm: If $n > 0$ is a rational number, then

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^n} = 0$$

↑
This theorem is based on limit laws

Q: Why a rational number? ←

ex3: $\lim_{x \rightarrow \infty} \frac{5x^3 + 2x - 7}{6x^3 + 3x^2 + 1}$

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 5x} - 2x)$$

ex4: $\lim_{x \rightarrow -\infty} \frac{(x + \sqrt{x^2 + 2x}) \cdot (x - \sqrt{x^2 + 2x})}{x - \sqrt{x^2 + 2x}}$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{x - \sqrt{x^2 + 2x}}$$

recall: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

so for $x \rightarrow -\infty$ we have $\sqrt{x^2} = |x| = -x$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{x - \sqrt{x^2(1 + \frac{2}{x})}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{x - |x| \sqrt{1 + \frac{2}{x}}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2}{1 + \sqrt{1 + \frac{2}{x}}}$$

More important examples

$$\lim_{x \rightarrow \pm\infty} e^x$$

$$\lim_{x \rightarrow \pm\infty} x^2$$

$$\lim_{x \rightarrow \pm\infty} x^3$$

ex 5: $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$

ex 6: Find a value of N s.t. $x > N \Rightarrow |f(x) - L| < \epsilon$

for the $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 1}}{x + 1} = 2$ and $\epsilon = 0.1$.

By graphing: we see N is @ least 18.86752.