2.1: The Tangent and Velocity Problem.

Concepts:

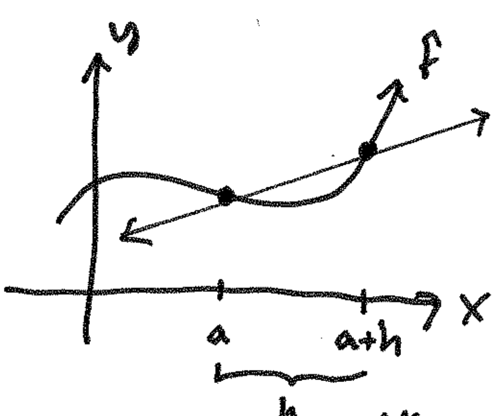
(1) Secant line: A line that intersects a curve more than once.

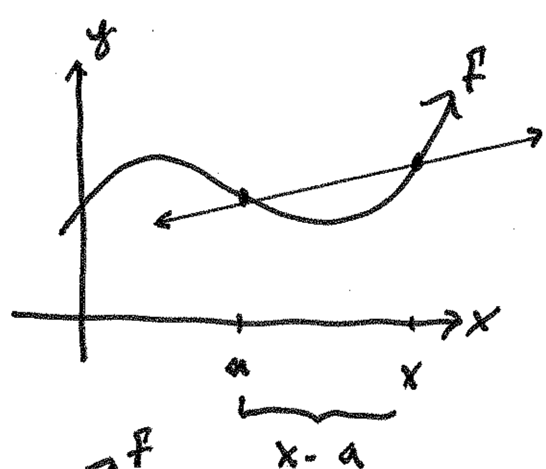
(2) Tangent line: A line w/ the same direction as a curve at the point of contact.

Mathamatica manipulate.

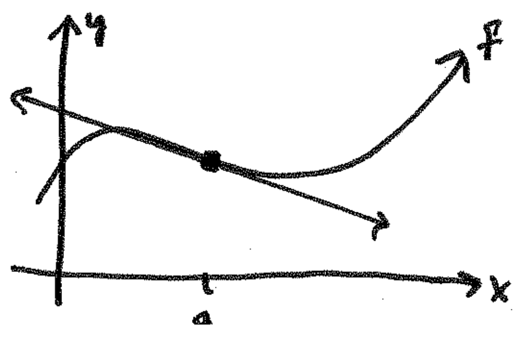
The picture clarifies that the tangent is the limit of the secants.

(1) secant

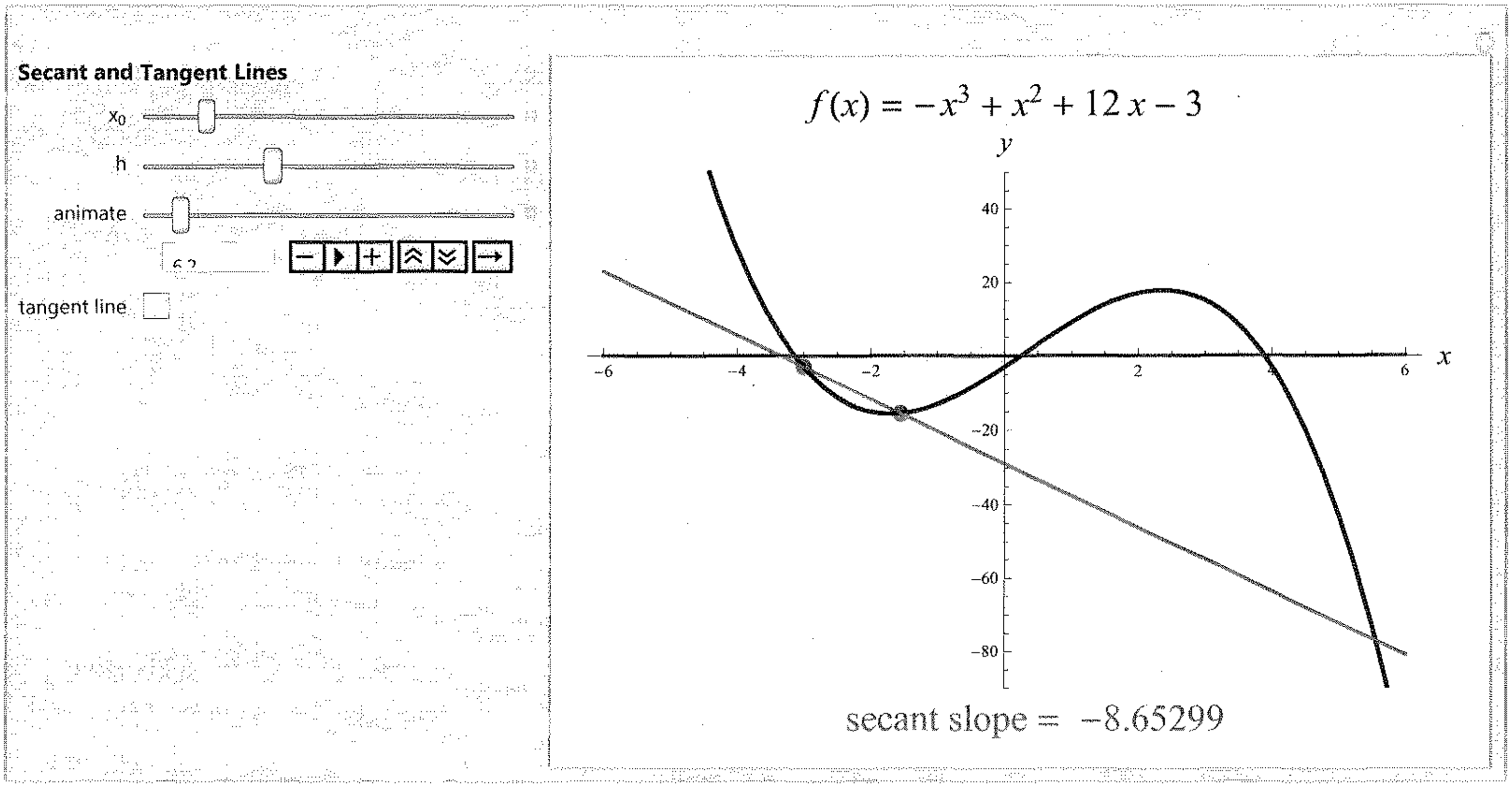




(2) tangent



Secant and Tangent Lines



recall: slope $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

slope of secant line : $\frac{f(a+h) - f(a)}{h}$

or : $\frac{f(x) - f(a)}{x - a}$

slope of tangent line: $\frac{f(a+h) - f(a)}{h}$ when $h \to 0$

$\frac{f(x) - f(a)}{x - a}$ when $x \to a$.

<u>Ex</u>: The pt $P(2,-1)$ lies on $f(x) = \frac{1}{1-x}$

(a) estimate the slope of the secant line PQ if Q is on f and includes
$x = 1.5, 1.9, 1.99$
$x = 2.5, 2.1, 2.01$

(b) estimate the slope of the tangent line to f @ P.

(c) Find the eqt. of the tangent line.

Notice that if a car is driving along a curve its head lights point along the tangent.
(matchbox car)

Velocity of a Falling Object

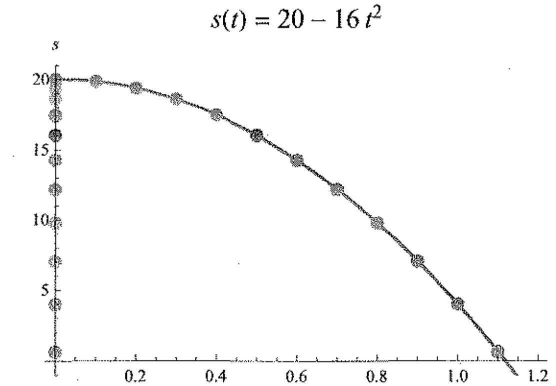
Average Rate of Change and Velocity

time t 0.5

t_0 0.2 0.6 1

time t	height s
0.0	20.0000
0.1	19.8400
0.2	19.3600
0.3	18.5600
0.4	17.4400
0.5	16.0000
0.6	14.2400
0.7	12.1600
0.8	9.7600
0.9	7.0400
1.0	4.0000
1.1	0.6400

$s(t) = 20 - 16\,t^2$



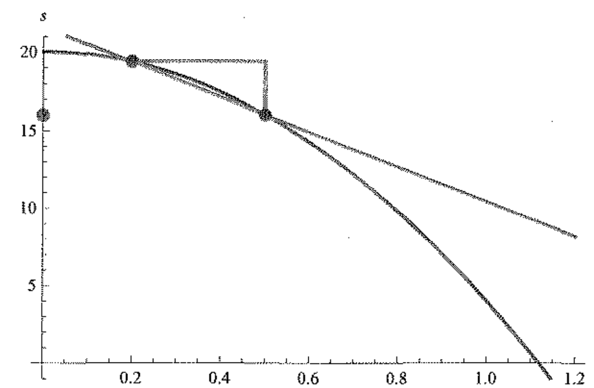
From time t to t_0:

$\Delta s = -3.3600$

$\Delta t = 0.3000$

average velocity

$\Delta s/\Delta t = -11.200$



2.1
4/15

ex: If a ball is dropped from a height of 20 ft, its ht t seconds later is given by $s(t) = 20 - 16t^2$

(a) Find the average velocity for the time period beginning at $t = 0.25$ and lasting 0.4, 0.1, and 0.01 seconds.

(b) estimate the inst. velocity when $t = 0.25$

[see Mathematica example]

There is a connection between tangent lines and local linearity.

ex: Explore the fcts & tangents thru zooming.

Fct	x	tangent
$y = x^2$	$x = 2$	$y = 4x - 4$
$y = x - 2\sin x$	$x = \frac{\pi}{2}$	$y = x - 2$