4.4: Indeterminate Form & L'Hospital's Rule

Ex1. $\lim\limits_{x \to 0} \frac{\sin x}{x}$

L'Hospital's Rule bought from Johann Bernoulli

L'Hospital's Rule

Suppose f & g are diff. & $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose

$\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ approaches the ind. form $\frac{0}{0}$

or $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ " " " " $\frac{\pm\infty}{\pm\infty}$.

Then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$ provided the limit on the RHS exists (or is $\pm\infty$).

Ex1 rev: $\lim\limits_{x \to 0} \frac{\sin x}{x} = \lim\limits_{x \to 0} \frac{\cos x}{1} = 1.$

Ex2: a) $\lim\limits_{x \to \infty} \frac{\ln \sqrt{x}}{x^2}$

b) $\lim\limits_{t \to 1} \frac{t^8 - 1}{t^5 - 1}$

c) $\lim\limits_{x \to 0} \frac{x - \sin x}{x - \tan x} = \lim\limits_{x \to 0} \frac{1 - \cos x}{1 - \sec^2 x} = \lim\limits_{x \to 0} \frac{\sin x}{-2\sec x \cdot \sec x \tan x} = \ldots$

Indeterminate Forms.

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $0\cdot\infty$ $\infty-\infty$ 0^0 ∞^0 1^∞

ex3: $\lim\limits_{x\to\infty} x\cdot\tan\left(\frac{1}{x}\right)$

ex4: $\lim\limits_{x\to 1} \frac{x}{x-1} - \frac{1}{\ln x}$

ex5: a) $\lim\limits_{x\to 0} (1-2x)^{1/x}$

b) $\lim\limits_{x\to\infty} \left(1+\frac{3}{x}\right)^{2x}$