2.8
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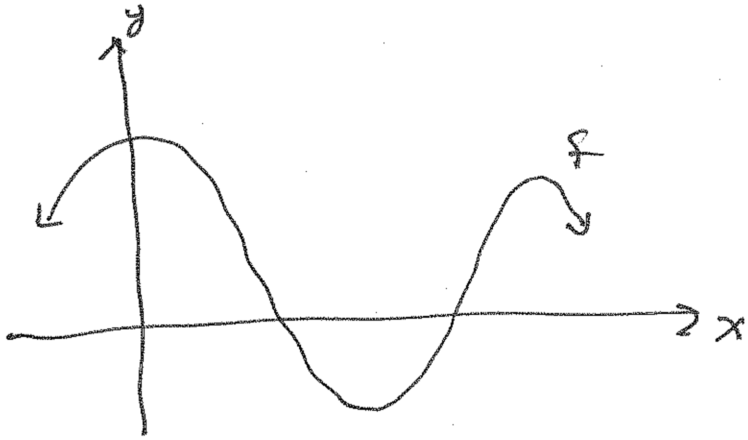
2.8: The Derivative of a Fcn

Dfn: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

ex1: Sketch the derivative given the graph of a fcn.



ex2: If $f(x) = x^4 + x$, find $f'(x)$. Then compare graphs...

ex3: If $g(x) = \sqrt{x}$, find $g'(x)$ & its domain.

Use $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$.

Other notations:

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx} f(x) = D. f(x) = D_x f(x).$$

$\frac{d}{dx}$ & D are differential operators.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x).$$

Leibnitz notation

Evaluation notation $\left.\frac{dy}{dx}\right|_{x=a}$.

Graph $g(x) = |x|$ & $g'(x)$....

<u>Thm</u>: If f is differentiable at a, then it is cont. @ a.

□ <u>proof</u>

Assume f is diff @ a.

$$\Rightarrow f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Now $f(x) - f(a) = \frac{f(x) - f(a)}{x - a} \cdot (x - a)$

and $\lim_{x \to a} (f(x) - f(a)) = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a} \cdot (x - a) \right]$

$$\Rightarrow \lim_{x \to a} [f(x) - f(a)] = f'(a) \cdot 0 = 0$$

$$\Rightarrow \lim_{x \to a} f(x) = \lim_{x \to a} \left[f(a) + (f(x) - f(a)) \right]$$

$$= \lim_{x \to a} f(a) + \lim_{x \to a} [f(x) - f(a)]$$

$$= f(a).$$

$\therefore$ f is cont. @ $x = a$. ∎

<u>NOTE</u>: The converse is not true

Ways a fct fails to be diff.

cusp, discont, or vert. tangent.

Notation for higher order deriv.