$\bar{X} = 78.4$ 6:29

med = 78.9 6:51

Test 2
Dusty Wilson
Math 220

Name: KEY

... we can repudiate completely and which we can abandon without regret because one does not know what this pretended sign signifies nor what sense one ought to attribute to it..

Augustin Cauchy
1789 – 1857 (French mathematician)

No work = no credit

Warm-ups (1 pt each)[1]: $\vec{e}_2 \cdot \vec{e}_2 =$ 1 $\quad \vec{e}_2^T \vec{e}_2 =$ 1 $\quad \vec{e}_1 \vec{e}_1^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

1.) (1 pt) The quote above gives Cauchy's understanding of $\sqrt{-1}$. Paraphrase Cauchy's sentiments (see above). Answer using complete English sentences.

Cauchy thought i was bunk.

2.) (10 pts) Show that the transformation $T(f(t)) = f''(4) + 4f'(t)$ from P_2 to P_2 is linear and determine if the transformation is an isomorphism (justify your answer).

$f(x) = ax^2 + bx + c \qquad f'(x) = 2ax + b \qquad f''(x) = 2a$

$g(x) = Lx^2 + mx + n \qquad g'(x) = 2Lx + m \qquad g''(x) = 2L$

$$T(f+g) = T\left((a+L)x^2 + (b+m)x + (c+n)\right)$$
$$= 2(a+L) + 4\left(2(a+L)x + (b+m)\right)$$
$$= \left(2a + 4(2ax+b)\right) + \left(2L + 4(2Lx+m)\right)$$
$$= T(f) + T(g)$$

$$T(kf) = T(kax^2 + kbx + kc)$$
$$= 2ka + 2kax + kb$$
$$= k(2a + 2ax + b)$$
$$= k\,T(f)$$

Hence T is a L.T.

It's not an isomorphism since ~~keeps~~ all constants are in the null space.

[1] In the warm-ups, $\vec{e}_i$ refers to the standard basis vector in $\mathbb{R}^2$.

3.) (10 pts) Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 6 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 4 & 9 & 1 & 2 \end{bmatrix}$

a.) Find a basis for the image of A.

$$\text{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 5 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -2x_2 - 5x_4$
x_2
$x_3 = x_4$
x_4
$x_5 = 0$

basis for $\text{im}(A)$: $\begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 9 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$

b.) Find the kernel of A.

$$\ker(A) = \text{span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right)$$

c.) rank(A) = 3 and nullity(A) = 2

4.) (4 pts) Answer the following:

a.) True or False, if vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ span $\mathbb{R}^n$, then $n = 4$

False

b.) True or False, if vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n \in \mathbb{R}^n$ are linearly independent, then $n = 4$

False.

5.) (10 pts) What conditions must be satisfied for a set V to be a subspace. Explain the meaning of any terms/phrases used in those conditions

(1) $\vec{0} \in V$

(2) V is closed under addition.

$\vec{u}, \vec{w} \in V \Rightarrow \vec{u} + \vec{w} \in V$

(3) V is closed under scalar mult.

$\vec{u} \in V$ and $k \in \mathbb{R} \Rightarrow k\vec{u} \in V$.

6.) (10 pts) Consider a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$. Prove that the image of T is a subspace.

(1) $T(\vec{0}) = \vec{0}$ (obvious).

(2) suppose $\vec{u}, \vec{v} \in im(T) \Rightarrow$ there exist $\vec{a}, \vec{b} \in \mathbb{R}^m$ s.t. $T(\vec{a}) = \vec{u}$ and $T(\vec{b}) = \vec{v}$.

$\Rightarrow T(\vec{a} + \vec{b}) = T(\vec{a}) + T(\vec{b})$

$= \vec{u} + \vec{v}$

so $\vec{u} + \vec{v}$ in $im(T)$

(3) suppose $\vec{u} \in im(T)$ and $k \in \mathbb{R}$

$\Rightarrow$ there exists $\vec{a} \in \mathbb{R}^m$ s.t. $T(\vec{a}) = \vec{u}$

$\Rightarrow T(k\vec{a}) = kT(\vec{a})$

$= k\vec{u}$

so $k\vec{u} \in im(T)$.

Hence T is a L.T.

4 pts - form
2 pts each cond.

at most 7/10 if im(A) are handled properly

7.) (10 pts) Find a basis for the space of all quadratics without a linear term.

Basis: $x^2, 1$ and dimension: 2

8.) (10 pts) Consider the linear transformation T such $T(\vec{v}_1) = -2\vec{v}_1 - 4\vec{v}_2$ and $T(\vec{v}_2) = -4\vec{v}_1 - 2\vec{v}_2$ where $\vec{v}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

a.) Find the matrix A of the linear transformation.

$$S = \begin{bmatrix} -1 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{tikzcd} \vec{x} \arrow[r, "A"] \arrow[d, "S^{-1}"'] & T(\vec{x}) \\ {[\vec{x}]_B} \arrow[r, "B"'] & {[T(\vec{x})]_B} \arrow[u, "S"'] \end{tikzcd}$$

$$B = \begin{bmatrix} -2 & -4 \\ -4 & -2 \end{bmatrix}$$

$$A = SBS^{-1} = \begin{bmatrix} -1 & +3 \\ 5 & -3 \end{bmatrix}$$

b.) If $\vec{x} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, find $T(\vec{x})$.

$$A\vec{x} = \begin{bmatrix} -1 & +3 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -18 \end{bmatrix}$$

(9.) similar

$$AS = \begin{bmatrix} 1 & -7 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} -1 & -4 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -15 & 17 \\ -14 & 10 \end{bmatrix} \checkmark$$

$$SA = \begin{bmatrix} -1 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} -15 & 17 \\ -14 & 10 \end{bmatrix} \checkmark$$