6.2: Properties of the Determinant

Thm: $\text{Det}(A^T) = \text{Det}(A)$

why? expand across row v. col.

How many calculations to evaluate an $n \times n$ determinant. ARGH

Use row ops...

(1) mult. by scalar. $\longrightarrow$ expand along row.

(2) row swap $\longrightarrow$ (a) sign changes when adj row swap.
(b) if non-adjacent...

(3) add rows. $\longrightarrow$ adding matrices.

what is the determinant of a matrix w/ two equal rows?

In summary

(1) If B is obtained from A by dividing a row of A by a scalar, then
$$\det(B) = \frac{1}{k}\det(A).$$

(2) If B is obtained from A by a row swap
$$\det(B) = -\det(A)$$

This means the determinant is ind. of basis

(3) If B is obtained from A by adding a mult. of a row of A to another row, then

$$\det(B) = \det(A)$$

Thus, if performing s row swaps & division of rows by scalars $k_1, \dots, k_r$ is required for rref of A, then

$$\det(\operatorname{rref}(A)) = (-1)^s \frac{1}{k_1 \cdots k_r} \det A$$

or $\det A = (-1)^s k_1 \cdots k_r \det(\operatorname{rref} A)$.

A is invertible iff $\operatorname{rref} A = I$ in which case $\det A = (-1)^s k_1 \cdots k_r \neq 0$.

<u>Thm</u>: If A & B are $n \times n$ matrices

$$\det(AB) = \det(A)\det(B)$$

<u>Thm</u>: If A is similar to B, then $\det A = \det B$

□ <u>proof</u>.

Suppose A & B are similar. This means that there is an invertible S s.t. ~~SA~~ $AS = SB$

$\Rightarrow \det(AS) = \det(SB)$

$\Rightarrow \det(A)\det(S) = \det(S)\det(B)$ where $\det(S) \neq 0$.

$\Rightarrow \det A = \det B$.

QED ■

Thm: If A is an invertible matrix, then $\det(A^{-1}) = \dfrac{1}{\det A}$.

□ proof.

Suppose A is invertible

$\Rightarrow I = A^{-1}A$

$\Rightarrow \det(I) = \det(A^{-1})\det(A)$

$\Rightarrow \det A^{-1} = \dfrac{1}{\det A}$

QED ■