# 1.1: Intro to Linear Systems

**ex1:** Consider the task of pulling a weight of 40 dan up a hill; we have one military horse, two ordinary horses, and three weak horses at our disposal. It turns out that the military horse and one of the ordinary horses, pulling together, are barely able to pull the weight (but no more). Likewise the two ordinary horses w/ one weak horse are just able to do the job, as are the three weak horses together w/ the military horse. How much weight can each of the horses pull together? (9 ch. ch 8 #12)

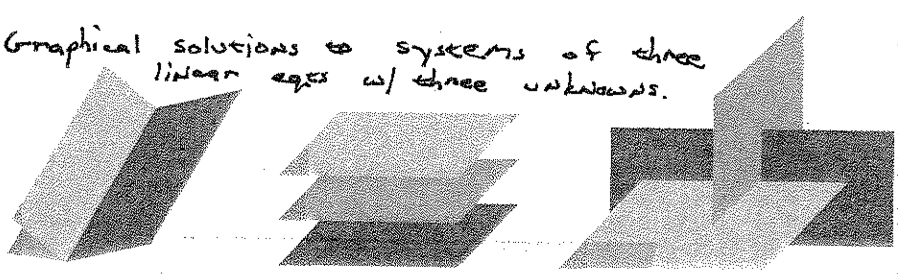
solution:

Let $m$ = dans a military horses can pull.
$\theta$ = dans a ordinary horses can pull
$w$ = dans a weak horses. can pull

$$\left|\begin{array}{rcl} m + \theta & = & 40 \\ 2\theta + w & = & 40 \\ m + 3w & = & 40 \end{array}\right| \quad E_3 - E_1 \to E_3$$

(see next page).

Graphical solutions to systems of three linear eqns w/ three unknowns.



$$\left|\begin{array}{rcrcrcl} m & + & \odot & & & = & 40 \\ & & 2\odot & + & w & = & 40 \\ m & + & & & 3w & = & 40 \end{array}\right| \quad \begin{array}{l} \\ \\ E_3 - E_1 \to E_3 \end{array}$$

$$\left|\begin{array}{rcrcrcl} m & + & \odot & & & = & 40 \\ & & 2\odot & + & w & = & 40 \\ & & -\odot & + & 3w & = & 0 \end{array}\right| \quad \begin{array}{l} \\ \frac{1}{2}E_2 \to E_2 \\ \\ \end{array}$$

$$\left|\begin{array}{rcrcrcl} m & + & \odot & & & = & 40 \\ & & \odot & + & \frac{1}{2}w & = & 20 \\ & & -\odot & + & 3w & = & 0 \end{array}\right| \quad \begin{array}{l} \\ \\ E_3 + E_2 \to E_3 \end{array}$$

$$\left|\begin{array}{rcrcrcl} m & + & \odot & & & = & 40 \\ & & \odot & + & \frac{1}{2}w & = & 20 \\ & & & & \frac{7}{2}w & = & 20 \end{array}\right| \quad \begin{array}{ll} E_1 - E_2 \to E_1 & (\text{new } E_2) \\ E_2 - \frac{1}{2}E_3 \to E_2 & (\text{new } E_3) \\ \frac{2}{7}E_3 \to E_3 & \end{array}$$

$$\left|\begin{array}{rcccl} m & & & = & \frac{160}{7} \\ & \odot & & = & \frac{120}{7} \\ & & w & = & \frac{40}{7} \end{array}\right|$$

The horses can pull $1\left(\frac{160}{7}\right) + 2\left(\frac{120}{7}\right) + 3\left(\frac{40}{7}\right)$

or $\frac{520}{7}$ dan when pulling together.

ex 2: Solve

$$\left|\begin{array}{l} x + 2y + 3z = 1 \\ x + 3y + 4z = 3 \\ x + 4y + 5z = 4 \end{array}\right| \begin{array}{l} \\ E_2 - E_1 \to E_2 \\ E_3 - E_1 \to E_3 \end{array}$$

$$\left|\begin{array}{l} x + 2y + 3z = 1 \\ 0 + y + z = 2 \\ 0 + 2y + 2z = 3 \end{array}\right| \begin{array}{l} \\ \\ E_3 - 2E_2 \to E_3 \end{array}$$

$$\left|\begin{array}{l} x + 2y + 3z = 1 \\ 0 + y + z = 2 \\ 0 + 0 + 0 = -1 \end{array}\right| \begin{array}{l} \\ \\ \Longleftarrow \quad 0 = -1 \quad \underline{\text{False}} \end{array}$$

No solution. (the system is "inconsistent").

ex3: After elimination, sometimes (many times) we are unable to find a single solution.

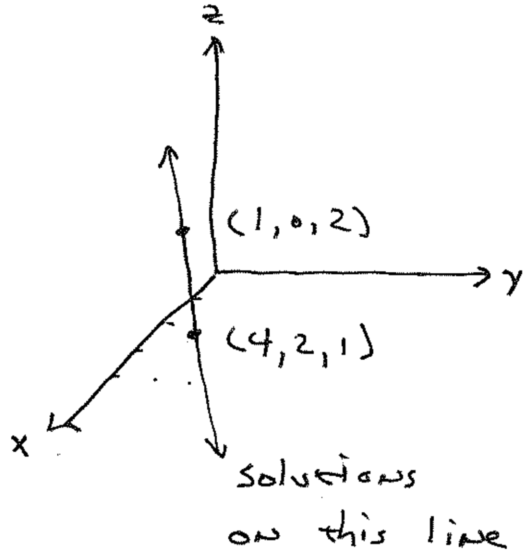
$$\left|\begin{array}{l} x + 0 + 3z = 7 \\ 0 + y + 2z = 4 \\ 0 + 0 + 0 = 0 \end{array}\right| \begin{array}{l} \\ \\ \Longleftarrow \quad 0 = 0 \quad \underline{\text{True}} \end{array}$$

$$\begin{array}{l} x + 3z = 7 \\ y + 2z = 4 \end{array} \quad \Rightarrow \quad \begin{array}{l} x = 7 - 3z \\ y = 4 - 2z \end{array}$$

2 eqns & 3 unknowns.

Notice that choosing values for $z$ determines points in the solution set.

$z=1 \Rightarrow (4,2,1)$ is a soln.




$(7-3t,\ 4-2t,\ t)$

This system is "consistent" and has infinitely many solutions.