1.3.63 This is the line parallel to $\vec{w}$ which goes through the end point of the vector $\vec{v}$.

1.3.64 This is the line segment connecting the head of the vector $\vec{v}$ to the head of the vector $\vec{v}+\vec{w}$.

1.3.65 This is the full parallelogram spanned by the two vectors $\vec{v}$ and $\vec{w}$.

1.3.66 Write $b = 1-a$ and $a\vec{v}+b\vec{w} = a\vec{v}+(1-a)\vec{w} = \vec{w}+a(\vec{v}-\vec{w})$ to see that this is the line segment connecting the head of the vector $\vec{v}$ to the head of the vector $\vec{w}$.

1.3.67 This is the full triangle with its vertices at the origin and at the heads of the vectors $\vec{v}$ and $\vec{w}$.

1.3.68 Writing $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$ as $\vec{u}\cdot(\vec{v}-\vec{w}) = 0$, we see that this is the line perpendicular to the vector $\vec{v}-\vec{w}$.

1.3.69 We write out the augmented matrix: $\begin{bmatrix} 0 & 1 & 1 & \vdots & a \\ 1 & 0 & 1 & \vdots & b \\ 1 & 1 & 0 & \vdots & c \end{bmatrix}$ and reduce it to $\begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{-a+b+c}{2} \\ 0 & 1 & 0 & \vdots & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \vdots & \frac{a+b-c}{2} \end{bmatrix}$.

So $x = \frac{-a+b+c}{2}$, $y = \frac{a-b+c}{2}$ and $z = \frac{a+b-c}{2}$.

1.3.70 We find it useful to let $s = x_1 + x_2 + \cdots + x_n$. Adding up all n equations of the system, and realizing that the term x_i is missing from the i^{th} equation, we see that $(n-1)s = b_1 + \cdots + b_n$, or, $s = \frac{b_1+\cdots+b_n}{n-1}$. Now the i^{th} equation of the system can be written as $s - x_i = b_i$, so that $x_i = s - b_i = \frac{b_1+\cdots+b_n}{n-1} - b_i$.

True or False

Ch 1.TF.**1** T, by Theorem 1.3.8

Ch 1.TF.**2** T, by Definition 1.3.9

Ch 1.TF.**3** T, by Definition.

Ch 1.TF.**4** F; Consider the equation $x+y+z=0$, repeated four times.

Ch 1.TF.**5** F, by Example 3a of Section 1.3

Ch 1.TF.**6** T, by Definition 1.3.7

Ch 1.TF.**7** T, by Theorem 1.3.4

Ch 1.TF.**8** F, by Theorem 1.3.1

Ch 1.TF.**9** F, by Theorem 1.3.4

Ch 1.TF.**10** F; As a counter-example, consider the zero matrix.

Ch 1.TF.11 T; The last component of the left-hand side is zero for all vectors $\vec{x}$.

Ch 1.TF.12 T; $A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \end{bmatrix}$, for example.

Ch 1.TF.13 T; Find rref

Ch 1.TF.14 T; Find rref

Ch 1.TF.15 F; Consider the 4×3 matrix A that contains all zeroes, except for a 1 in the lower left corner.

Ch 1.TF.16 F; Note that $A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2A \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for all 2×2 matrices A.

Ch 1.TF.17 F; The rank is 1.

Ch 1.TF.18 F; The product on the left-hand side has two components.

Ch 1.TF.19 T; Let $A = \begin{bmatrix} -3 & 0 \\ -5 & 0 \\ -7 & 0 \end{bmatrix}$, for example.

Ch 1.TF.20 T; We have $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$.

Ch 1.TF.21 F; Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$, for example.

Ch 1.TF.22 T, by Exercise 1.3.44.

Ch 1.TF.23 F; Find rref to see that the rank is always 2.

Ch 1.TF.24 T; Note that $\vec{v} = 1\vec{v} + 0\vec{w}$.

Ch 1.TF.25 F; Let $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example.

Ch 1.TF.26 T; Note that $\vec{0} = 0\vec{v} + 0\vec{w}$

Ch 1.TF.27 F; Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, for example. We can apply elementary row operations to A all we want, we will always end up with a matrix that has all zeros in the first column.

Ch 1.TF.28 T; If $\vec{u} = a\vec{v} + b\vec{w}$ and $\vec{v} = c\vec{p} + d\vec{q} + e\vec{r}$, then $\vec{u} = ac\vec{p} + ad\vec{q} + ae\vec{r} + b\vec{w}$.

Ch 1.TF.29 F; The system $x = 2,\ y = 3,\ x + y = 5$ has a unique solution.

Ch 1.TF.30 F; Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example.

Ch 1.TF.31 F; If $A\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \vec{0}$, then $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a solution to $\left[A \vdots \vec{0}\right]$. However, since rank$(A) = 3$, rref$\left[A \vdots \vec{0}\right] =$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 \vdots 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, meaning that only $\vec{0}$ is a solution to $A\vec{x} = \vec{0}$.

Ch 1.TF.32 F; If $\vec{b} = \vec{0}$, then having a row of zeroes in rref(A) does not force the system to be inconsistent.

Ch 1.TF.33 T; By Example 4d of Section 1.3, the equation $A\vec{x} = \vec{0}$ has the unique solution $\vec{x} = \vec{0}$. Now note that $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and $\vec{v} = \vec{w}$.

Ch 1.TF.34 T; Note that rank$(A) = 4$, by Theorem 1.3.4

Ch 1.TF.35 F; Let $\vec{u} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example..

Ch 1.TF.36 T; We use rref to solve the system $A\vec{x} = \vec{0}$ and find $\vec{x} = \begin{bmatrix} -2t \\ -3t \\ t \end{bmatrix}$, where t is an arbitrary constant.

Letting $t = 1$, we find $[\vec{u}\ \vec{v}\ \vec{w}]\begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} = -2\vec{u} - 3\vec{v} + \vec{w} = \vec{0}$, so that $\vec{w} = 2\vec{u} + 3\vec{v}$.

Ch 1.TF.37 F; Let $A = B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, for example.

Ch 1.TF.38 T; Matrices A and B can both be transformed into $I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Running the elementary operations backwards, we can transform I into B. Thus we can first transform A into I and then I into B.

Ch 1.TF.39 T; If $\vec{v} = a\vec{u} + b\vec{w}$, then $A\vec{v} = A(a\vec{u} + b\vec{w}) = A(a\vec{u}) + A(b\vec{w}) = aA\vec{v} + bA\vec{w}$.

Ch 1.TF.40 T; check that the three defining properties of a matrix in rref still hold. F; If $\vec{b} = \vec{0}$, then having a row of zeroes in rref(A) does not force the system to be inconsistent.

Ch 1.TF.41 T; $A\vec{x} = \vec{b}$ is inconsistent if and only if rank$\left[A \vdots \vec{b}\right] =$ rank$(A)+1$, since there will be an extra leading one in the last column of the augmented matrix: (See Figure 1.16.)

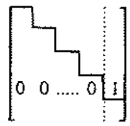

Figure 1.16: for Problem T/F 41.

Ch 1.TF.**42** T; The system $A\vec{x} = \vec{b}$ is consistent, by Example 4b, and there are, in fact, infinitely many solutions, by Example 4c. Note that $A\vec{x} = \vec{b}$ is a system of three equations with four unknowns.

Ch 1.TF.**43** T; Recall that we use $\text{rref}\left[A \vdots \vec{0}\right]$ to solve the system $A\vec{x} = \vec{0}$. Now, $\text{rref}\left[A \vdots \vec{0}\right] = \left[\text{rref}(A) \vdots \vec{0}\right] = \left[\text{rref}(B) \vdots \vec{0}\right] = \text{rref}\left[B \vdots \vec{0}\right]$. Then, since $\left[\text{rref}(A) \vdots \vec{0}\right] = \left[\text{rref}(B) \vdots \vec{0}\right]$, they must have the same solutions.

Ch 1.TF.**44** F; Consider $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. If we remove the first column, then the remaining matrix fails to be in rref.

Ch 1.TF.**45** T; First we list all possible matrices rref(M), where M is a 2×2 matrix, and show the corresponding solutions for $M\vec{x} = \vec{0}$:

rref(M)	solutions of $M\vec{x} = \vec{0}$
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\{\vec{0}\}$
$\begin{bmatrix} 1 & a \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -at \\ t \end{bmatrix}$, for an arbitrary t
$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} t \\ 0 \end{bmatrix}$, for an arbitrary t
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\mathbb{R}^2$

Now, we see that if $\text{rref}(A) \neq \text{rref}(B)$, then the systems $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ must have different solutions. Thus, it must be that if the two systems have the same solutions, then $\text{rref}(A) = \text{rref}(B)$.

Ch 1.TF.**46** T . First note that the product of the diagonal entries is nonzero if (and only if) all three diagonal entries are nonzero.

If all the diagonal entries are nonzero, then $A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{matrix} \div a \\ \div c \\ \div f \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ b' & 1 & 0 \\ d' & e' & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, showing that rank $A = 3$.

Conversely, if $a = 0$ or $c = 0$ or $f = 0$, then it is easy to verify that rref A will contain a row of zeros, so that rank $A \leq 2$. For example, if a and c are nonzero but $f = 0$, then

$A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & 0 \end{bmatrix} \begin{matrix} \div a \\ \div c \\ \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ b' & 1 & 0 \\ d & e & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, with rank $A = 2$.

Ch 1.TF.47 T. If $a \neq 0$, then $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \div a \quad \rightarrow \quad \begin{bmatrix} 1 & b/a \\ c & d \end{bmatrix} -c(I) \quad \rightarrow \quad \begin{bmatrix} 1 & b/a \\ 0 & (ad-bc)/a \end{bmatrix} \; a/(ad-bc)$

$\rightarrow \quad \begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, showing that rank $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 2$.

If $a = 0$, then b and c are both nonzero, so that $\begin{bmatrix} 0 & b \\ c & d \end{bmatrix}$ reduces to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ as claimed.

Ch 1.TF.48 T. If $\vec{w} = a\vec{u} + b\vec{v}$, then $\vec{u} + \vec{v} + \vec{w} = (a+1)\vec{u} + (b+1)\vec{v} = (a-b)\vec{u} + (b+1)(\vec{u} + \vec{v})$

Ch 1.TF.49 T. If $A\vec{v} = \vec{b}$ and $A\vec{w} = \vec{c}$, then $A(\vec{v} + \vec{w}) = \vec{b} + \vec{c}$, showing that the system $A\vec{x} = \vec{b} + \vec{c}$ is consistent. Suppose A is an $n \times m$ matrix. Since $A\vec{x} = \vec{b}$ has a unique solution, rank A must be m (by Example 1.3.3c), implying that the system $A\vec{x} = \vec{b} + \vec{c}$ has a unique solution as well (by Example 1.3.4d).

Ch 1.TF.50 F. Think about constructing a 0-1 matrix A of size 3×3 with rank $A = 3$ row by row. The rows must be chosen so that rref A will not contain a row of zeros, which implies that no two rows of A can be equal. For the first row we have $7 = 2^3 - 1$ choices: anything except $[\; 0 \quad 0 \quad 0 \;]$. For the second row we have six choices left: anything except the row of zeros and the first row. For the third row we have at most five choices, since we cannot chose the row of zeros, the first row, or the second row. Thus, at most $7 \times 6 \times 5 = 210$ of the 0-1-matrices of size 3×3 have rank 3, out of a total of $2^9 = 512$ matrices.