Test 3
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Math 111

Name: Key

No work = no credit

No Symbolic Calculators

Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers ... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.

John Napier (1550 - 1617)
Scottish mathematician

Warm-ups (1 pt each): $-2^2 = -4$ $\log_3(3) = 1$ $-e^0 = -1$

1.) (1 pt) Based upon the quote above, why did Napier invent the logarithm? Answer using complete English sentences.

Logs make life easier.

2.) (4 pts) Solve $\frac{3x}{4} - \frac{1}{3} = 1 - \frac{2}{3}\left(x - \frac{1}{6}\right)$ $\Rightarrow 51x = 52$

$\rightarrow \frac{3x}{4} - \frac{1}{3} = 1 - \frac{2}{3}x + \frac{2}{18}$

$\Rightarrow 27x - 12 = 36 - 24x + 4$ $x = \frac{52}{51}$

3.) (2 pts) Is $e = \frac{260412269}{95800320}$? Explain

NO, e is irrational

4.) (4 pts) The population of Ethiopia was 90.2 million in 2014 and growing by 2.1% annually. Set up an exponential model describing the population and use it to <u>algebraically</u> determine the year when the population of Ethiopia will reach 100 million.

Solve $100 = 90.2(1.021)^t$

$\frac{100}{90.2} = 1.021^t$

$t = \frac{\ln\left(\frac{100}{90.2}\right)}{\ln(1.021)}$

$= 4.96$

Model: $P(t) = 90.2(1.021)^t$

Year: 2019

5.) (4 pts) Write the expression $3\log_4 x+\frac{1}{2}\log_4(x+1)$ as a single logarithm.

$\log_4(x^3\sqrt{x+1})$

6.) (2 pts) If $\log_b(5)=u$ and $\log_b 7=v$, find the value of $\log_b\left(\frac{7}{5}\right)$ in terms of *u* and *v*.

$v-u$

7.) (4 pts) Find the sum of the first 200 terms of the geometric sequence 5, 15, 45, ...

$a_1=5$

$r=3$

$S_{200}=\frac{5(3^{200}-1)}{3-1}$

8.) (4 pts) Adah invested $10,000 at 6% compounded continuously. What is the value of the investment after 12 years? Answer using a complete sentence.

$FV=10000e^{0.06(12)}$

$=20544.33$

Adah will have $20,544.33

9.) (4 pts) How much must Bartholomew invest today at 8% compounded semiannually to have $30,000 in 18 years? Answer using a complete sentence.

N= 2·18
I%= 8
PV= 7310.06
PMT= 0
FV= 30000
P/Y= 2
C/Y= 2

Bartholomew must save $7310.06 today.

10.) (4 pts) Magdalene saves $100 a month. What interest rate (compounded monthly) must she receive to save $10,000 in 6 years? Answer using a complete sentence.

N= 6·12
I%= 10.6
PV= 0
PMT= 100
FV= -10000
P/Y= 12
C/Y= 12

Magdalene must earn 10.6% interest.

11.) (4 pts) Tabitha borrowed $200,000 at 3%. She paid the loan off after 25 years of monthly payments. How much interest did she pay over the life of the loan? Answer using a complete sentence.

N= 12(25)
I%= 3
PV= 200000
PMT= 948.42
FV= 0
P/Y= 12
C/Y= 12

total payout: 284526
- 200000
interest: $84526

12.) (4 pts) Zipporah invests $250 at the end of each month beginning at the age of 20. After 45 years of contributions, she retires and stops making deposits. At this point she begins to withdraw $15,000 each month. How old will she be when she completely depletes her savings ($0 left)? Assume a constant rate of 9%, compounded monthly? Answer using a complete sentence.

N= 45(12)	*	N= 347.75 ≈ 29 yrs.
I%= 9		I%= 9
PV= 0		PV= 18912.19.62
PMT= 250		PMT= -15000
* FV= 1851219.62		FV= 0
P/Y= 12		P/Y= 12
C/Y= 12		C/Y= 12

20
+45
+29
94

Zipporah will be 94 years old.

13.) (4 pts) Erastus borrowed $170,000 at 4% for 30 years. After 5 years of monthly payments, he refinanced the balance at 3% for 20 years. What was total amount Erastus paid over the life of the loans? Answer using a complete sentence.

N= 30(12)		N= 20(12)
I%= 4		I%= 3
PV= 170000		PV= 153760.77
PMT= -811.61	*	PMT= 852.75
FV= 0		FV= 0
P/Y= 12		P/Y= 12
C/Y= 12		C/Y= 12

balance in 5 years.
153760.77 ← 3pts.

Total paid 60(811.61)
+ 240(852.75)
253356.60

Erastus paid a total of $253,356.60 for his $170,000 house.