Test 2
Dusty Wilson
Math 111

Name: _____ key _____

*Madam, I have come from a country where
people are hanged if they talk.*

Leonard Euler (1707 - 1783)
Swiss mathematician

No work = no credit

No Symbolic Calculators

Warm-ups (1 pt each): $e^0 = \underline{1}$ $-3^2 = \underline{-9}$ $\sqrt{-4} = \dfrac{2i \ \text{not real}}{\text{undefined}}$

1.) (1 pt) In the quote above, Leonard Euler excuses his taciturnity in conversation to the Queen Mother of Prussia following his return from Russia. Based upon the quote above, was free speech encouraged in 18th century Russia? Answer using complete English sentences.

No, saying too much could stretch your neck

2.) (4 pts) Find the exact solution to: $\dfrac{3x}{4} - \dfrac{1}{3} = 1 - \dfrac{2}{3}\left(x - \dfrac{1}{6}\right)$

$\dfrac{3x}{4} - \dfrac{1}{3} = 1 - \dfrac{2}{3}x + \dfrac{2}{18}$

$\Rightarrow 27x - 12 = 36 - 24x + 4$

$\Rightarrow 51x = 52$

$\Rightarrow x = \dfrac{52}{51}$

3.) (8 pts) <u>Algebraically</u> find the vertex and x and y intercepts of $y = x^2 + 2x - 15$. Show your work.

$x = -\dfrac{2}{2(1)} = -1$

$y = 1 - 2 - 15$

$\underline{(-1, -16)}$
vertex

$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(-15)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{64}}{2}$

$= -1 \pm 4$

$\underline{x = -5 \text{ or } x = 3}$
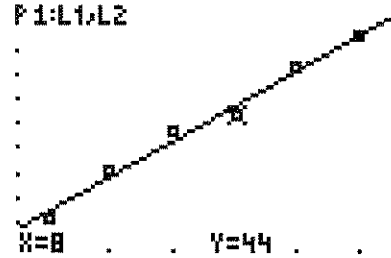x intercept(s)

$\underline{y = -15}$
y intercept(s)

4.) (2 pts) Consider the data and model shown in the given picture. The model is $y = 5x + 7$. What is the residual when $x = 8$? Hint: Is the residual positive or negative?

Note: the given data point is (8,44)

$$Actual = 44$$

$$Approx = y'(8) \approx 47$$

$$Residual = 44 - 47 = -3$$

$$p(x) = y'$$

P1:L1,L2

X=8   Y=44

5.) (4 pts) The table gives the average gas price in Seattle where x is given in months since January 2014 (January 2014 is $x = 0$). Data is taken from a chart on gasbuddy.com

Find a quadratic model $f(x)$ for the data where x is given in months since January 2014. Give your answer to three decimal places.

$$f(x) \approx -0.032x^2 + 0.362x + 2.979$$

Month	Gas Price
1	$3.34
3	$3.74
5	$3.97
7	$3.96
9	$3.71
11	$3.10
13	$2.30

   a.) Find and interpret $f(12) = 2.74$

Gas cost about $2.74 in Jan. 2015

6.) (4 pts) Solve $5x^2 = 2x + 6$ using any algebraic method. Give the <u>exact</u> answer(s).

$$\Rightarrow 5x^2 - 2x - 6 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 - 4(5)(-6)}}{2(5)}$$
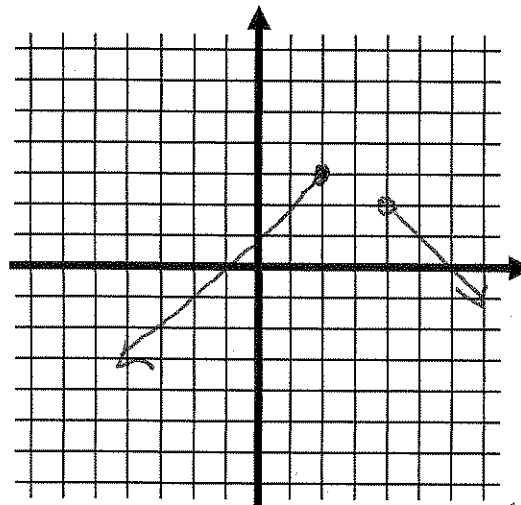
$$= \frac{2 \pm \sqrt{124}}{10}$$

$$= \frac{1 \pm \sqrt{31}}{5}$$

7.) (6 pts) Consider $g(x) = \begin{cases} x+1, & x \le 2 \\ 6-x, & x > 4 \end{cases}$

a.) Evaluate $g(5)$

$$g(5) = 1$$

b.) Carefully sketch a graph of $g(x)$

c.) What is the domain of g?

$$(-\infty, 2] \cup (4, \infty)$$

d.) What is the range of g?

$$(-\infty, 3]$$

graph, ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ II 2a-yes
28 - no.

8.) (2 pts) Suppose you want to model the total cost of renting a new 2 bedroom apartment.

- There is a non-refundable $500 deposit.
- Then you sign a six month lease to pay $1000 per month in rent.
- After six months, your rent goes up to $1050/mo.

Set up a piecewise defined function to model the total cost of the apartment where x is given in months since you moved in.

$$\text{Total Rent}(x) = \begin{cases} 500 + 1000x & , \ 0 \le x \le 6 \\ 500 + 6000 + 1050(x-6) & , \ x > 6 \end{cases}$$

~~IIII~~ ~~IIII~~ II   1.5 - 2 — 12
~~IIII~~ ~~IIII~~ III   1   — 14
~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ I   0 - 0.5 — 19 26

9.) (8 pts) Suppose a company has fixed costs of \$300 and variable costs of $\frac{3}{4}x + 1460$ dollars per unit, where x is the total number of units produced. Suppose further that the selling price of its product is $1500 - \frac{1}{4}x$ dollars per unit.

a.) Formulate the functions for total cost, revenue, and profit.

$$C(x) = \left(\frac{3}{4}x + 1460\right)x + 300 = \frac{3}{4}x^2 + 1460x + 300$$

$$R(x) = \left(1500 - \frac{1}{4}x\right)x = 1500x - \frac{1}{4}x^2$$

$$P(x) = 1500x - \frac{1}{4}x^2 - \left(\frac{3}{4}x^2 + 1460x + 300\right) = -x^2 + 40x - 300$$

b.) <u>Algebraically</u> find and <u>interpret</u> the break even points.

Solve $0 = -x^2 + 40x - 300$

$= x^2 - 40x + 300$

$= (x - 30)(x - 10)$

$\Rightarrow x = 30 \text{ or } x = 10$

The company breaks even w/ the sale of 10 or 30 units.

c.) <u>Algebraically</u> find and <u>interpret</u> the vertex of the profit function.

$$X = -\frac{40}{2(-1)} = 20$$

$$P(20) = 100$$

Profit is maxed @ \$100 w/ the sale of 20 units.

d.) Find and <u>interpret</u> the selling price for the item when profit is maximized.

$$price = 1500 - \frac{1}{4}(20) = 1495$$

sell items @ \$1495 ea. to max profit.