No work = no credit

No Symbolic Calculators

Name: ___Key.___

Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers ... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.

John Napier (1550 - 1617)
Scottish mathematician

Warm-ups (1 pt each): $-5^2 = \underline{-25}$ $\log_5(1) = \underline{0}$ $\pi^0 = \underline{1}$

1.) (1 pt) Based upon the quote above, why did Napier invent the logarithm? Answer using complete English sentences.

To make life easier.

2.) (4 pts) Solve $\dfrac{3x}{4} - \dfrac{1}{3} = 1 - \dfrac{2}{3}\left(x - \dfrac{1}{6}\right)$

$\Rightarrow \dfrac{3x}{4} - \dfrac{1}{3} = 1 - \dfrac{2}{3}x + \dfrac{2}{18}$

$LCD = 36$

$\Rightarrow 27x - 12 = 36 - 24x + 4$

$\Rightarrow 51x = 52$

$x = \dfrac{52}{51}$

3.) (4 pts) Write $4^3 = 64$ in logarithmic form.

$\log_4(64) = 3$

4.) (4 pts) Write the expression $\log_5\left(\dfrac{\sqrt{x}}{(x+1)^8}\right)$ as the sum or difference of logs without exponents.

$= \log_5(\sqrt{x}) - \log_5((x+1)^8)$

$= \dfrac{1}{2}\log_5(x) - 8\log_5(x+1)$

5.) (6 pts) The table below shows Kenya's estimated population.

Year	Population (millions)
2004	34
2014	45

Source: www.wolframalpha.com

a.) Find the exponential model for the data $y = a * b^x$ and explain the meaning of a and b using everyday language. Make $x = 0$ represent 2004. Round b to four decimal places.

$$x = 0: \quad 34 = a \cdot b^0 \implies a = 34$$

$$x = 10: \quad 45 = 34 \cdot b^{10} \implies \frac{45}{34} = b^{10} \implies \left(\frac{45}{34}\right)^{1/10} = b$$

$$\text{So} \quad b = 1.0284$$

$$y = 34 \cdot 1.0284^x$$

The pop in 2004 was 34 mil.

The pop grows 2.84% annually

b.) Evaluate and interpret your model when $x = 15$. Does this make sense?

The model predicts a pop. of 51.75 mil in 2019. This seems reasonable.

6.) (4 pts) Find the sum of the first 200 terms of the arithmetic sequence that begins 12, 9, 6, ...

$$a_n = 12 - 3(n-1)$$

$$a_{200} = 12 - 3(199) = -585$$

$$S_{200} = \frac{200}{2}(12 + (-585))$$

$$\underline{-57300}$$

7.) (4 pts) Write $\frac{1}{2}\ln(x+3) + 5\ln(x-1)$ as a single logarithm.

$$\underline{\ln\left[\sqrt{x+3} \cdot (x-1)^5\right]}$$

T4

8.) (5 pts) Dusty invested $5000 at 8% compounded *continuously*. How long will it take his investment to double? Solve algebraically and answer using a complete sentence.

$$10000 = 5000 e^{0.08t}$$

$$\Rightarrow 2 = e^{0.08t}$$

$$\Rightarrow \ln 2 = 0.08t$$

$$\Rightarrow t = \frac{\ln 2}{0.08} = 8.66$$

It will take 8.66 yrs for the investment to double.

9.) (5 pts) How much must Charlene invest today at 5% compounded *semiannually* to have $10,000 in 7 years? Answer using a complete sentence.

```
N= 2·7=14
I%= 5
✳ PV= 7077.27
PMT= 0
FV= 10000
P/Y= 2
```

Chan should invest $7077.27 today.

$\frac{2}{5}$ if payments.

10.) (5 pts) Melissa saves $25 a week (52 weeks a year). What interest rate (compounded *weekly*) must she receive to save $10,000 in 5 years? Answer using a complete sentence.

```
N= 5·52=260
✳ I%= 16.2
PV= 0
PMT= 25
FV= -10,000
P/Y= 52
```

She must earn 16.2% interest, compounded weekly.

15

11.) (5 pts) Faith borrowed $150,000 at 4%. She paid the loan off after 30 years of monthly payments. How much interest did she pay over the life of the loan? Answer using a complete sentence.

N= 12·30 =360
I%= 4
PV= 150000
PMT= 716.12
FV= 0
P/Y= 12

✳

Total paid: 257,803.20

She paid $107803.2 in interest.

12.) (5 pts) Dustin invests $200 at the end of each month beginning at the age of 18. After 20 years of contributions, he stops making deposits and just leaves the money in the account. How old will Dustin be when the account is worth $2,000,000? The investment earn a constant rate of 8%, compounded monthly. Answer using a complete sentence.

N= 20·12 =240 ✳	N= 426.2 ←
I%= 8	I%= 8
PV= 0	PV= 117804.08
PMT= 200	PMT= 0
FV= 117,804.08	FV= -2,000,000
P/Y= 12	P/Y= 12

✳✳

35.5 yrs

18
+20
+35.5
———
73.5

Dust will be 73.5 y.o.

13.) (5 pts) Mooie borrowed $240,000 at 7% for 30 years. After 10 years of monthly payments, he refinanced the balance at 4% for 20 years. How much money did Mooie save by refinancing? (Note: We are ignoring fees and other costs)

N= 360	N= 240
I%= 7	I%= 4
PV= 240000	PV= 205949.72
PMT= 1596.73 ✳	PMT= 1248.01
FV= 0	FV= 0
P/Y= 12	P/Y= 12

✳

Total cost w/ refi
120(1596.73) + 240(1248.01)
= $491130

Moo saved $83692.80.

w/o refi, Moo would pay $574,822.80
balance @ time of refi (p=120) $205949.72

15