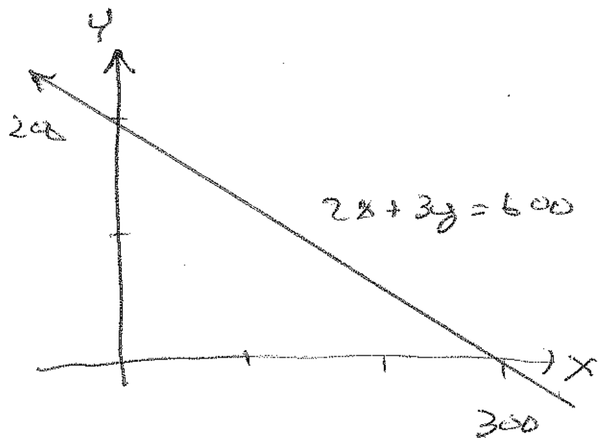
Graphing Lines.

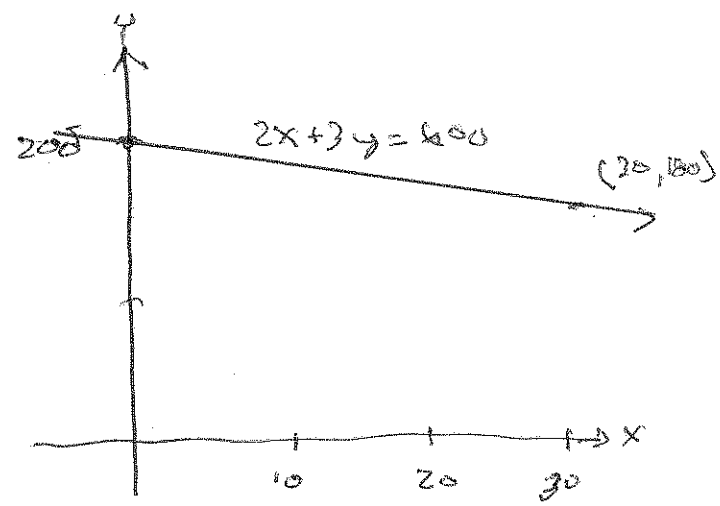
$2x + 3y = 600$

x	y
0	200
300	0



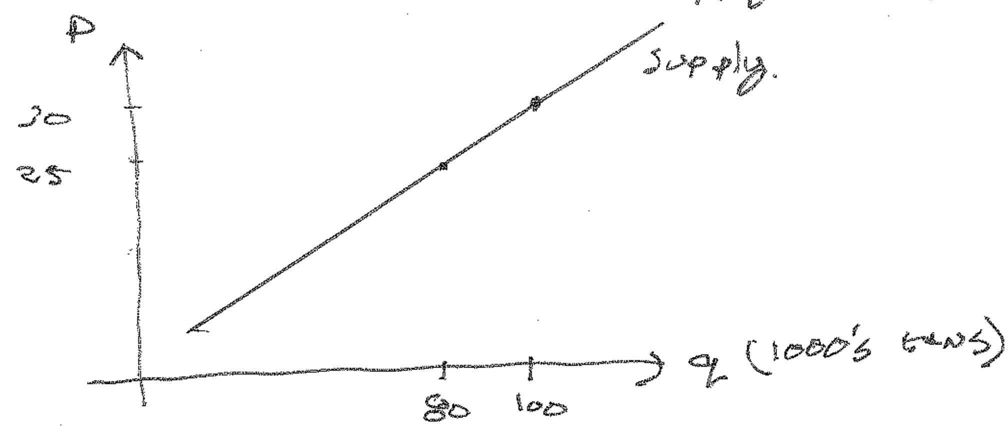
x	y
0	200
300	0
30	180

same line...
different scale.



Supply & Demand. (1.6 #38)

Suppose a mining co. will supply 100K tons @ \$30/ton & 80k tons @ \$25/ton. Write the supply model.



$\text{slope} = \frac{5}{20} = \frac{1}{4}$

$\Rightarrow p - 30 = \frac{1}{4}(q - 100)$

$\Rightarrow p = \frac{1}{4}q - 25 + 30$

$\Rightarrow p = \frac{1}{4}q + 5$

<u>interpret</u>

→ p-int. $(p = 5)$

No one will be supplied @ \$5/ton.

→ slope $(m = 1/4)$

for each additional 1000 tons supplied, the price/ton increases \$0.25.

Supply & Demand (1.6 #48)

(q, p)

Retailers will buy 45 @ \$10/ea. $(45, 10)$ } Demand

20 @ \$60/ea $(20, 60)$

Wholesalers will supply 35 @ \$30/ea $(35, 30)$ } Supply

70 @ \$50/ea. $(70, 50)$

Find - supply & demand & market equilibrium.

<u>Demand curve.</u>

slope: $m = \frac{60-10}{20-45} = \frac{50}{-25} = -2$

line: $p - 10 = -2(q - 45)$

$\Rightarrow p - 10 = -2q + 90$

$\Rightarrow D: p = -2q + 100$

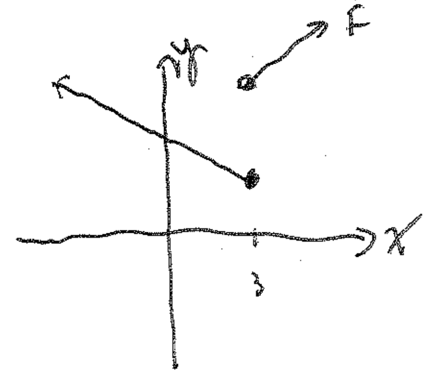
<u>Supply curve</u>

$S: p = \frac{45}{96}q + 10$

on the calculator we find $q = 35$ & $p = 30$

Market equilibrium is reached when 35 units are sold @ \$30/ea.

Piecewise defined fcts.



$$f(x) = \begin{cases} 2x+1, & x > 3 \\ 4-x, & x \le 3 \end{cases}$$

$y1 = (2x+1)/(x>3)$

$y2 = (4-x)/(x \le 3)$

use $\boxed{2nd} \to \boxed{Test}$ to find inequalities.

Piecewise defined Fcts.

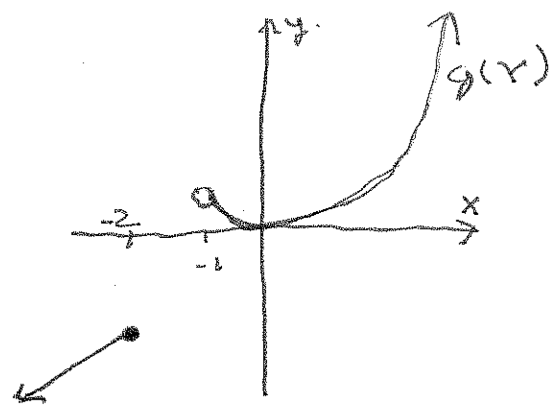
$$g(x) = \begin{cases} x^2, & x > -1 \\ 2x+1, & x \leq -2 \end{cases}$$

$g(4) = 16$; $g(-4) = -7$; $g(-1.5)$ is undefined.

$y1 = x^2 / (x > -1)$

$y2 = (2x+1) / (x \leq -2)$

$\boxed{2nd} \to \boxed{Test}$



Gauss Jordan Elimination.

$$\begin{cases} x + 2y = 3 \\ 4x + 5y = 6 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right] \quad 4R_1 - R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 3 & 6 \end{array}\right] \quad \frac{1}{3}R_2 \to R_2$$

$$\left[\begin{array}{ccc} 1 & 2 & 3 \\ 0 & 1 & 2 \end{array}\right] \quad -2R_2 + R_1 \to R_1$$

$$\left[\begin{array}{cc|c} +1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$$

$x = -1; \; y = 2$

Gauss- Jordan Elimination.

$$\left[\begin{array}{cc:c} 3 & 9 & 72 \\ 9 & 2 & 2 \end{array}\right] \quad \frac{1}{3}R_1 \rightarrow R_1$$

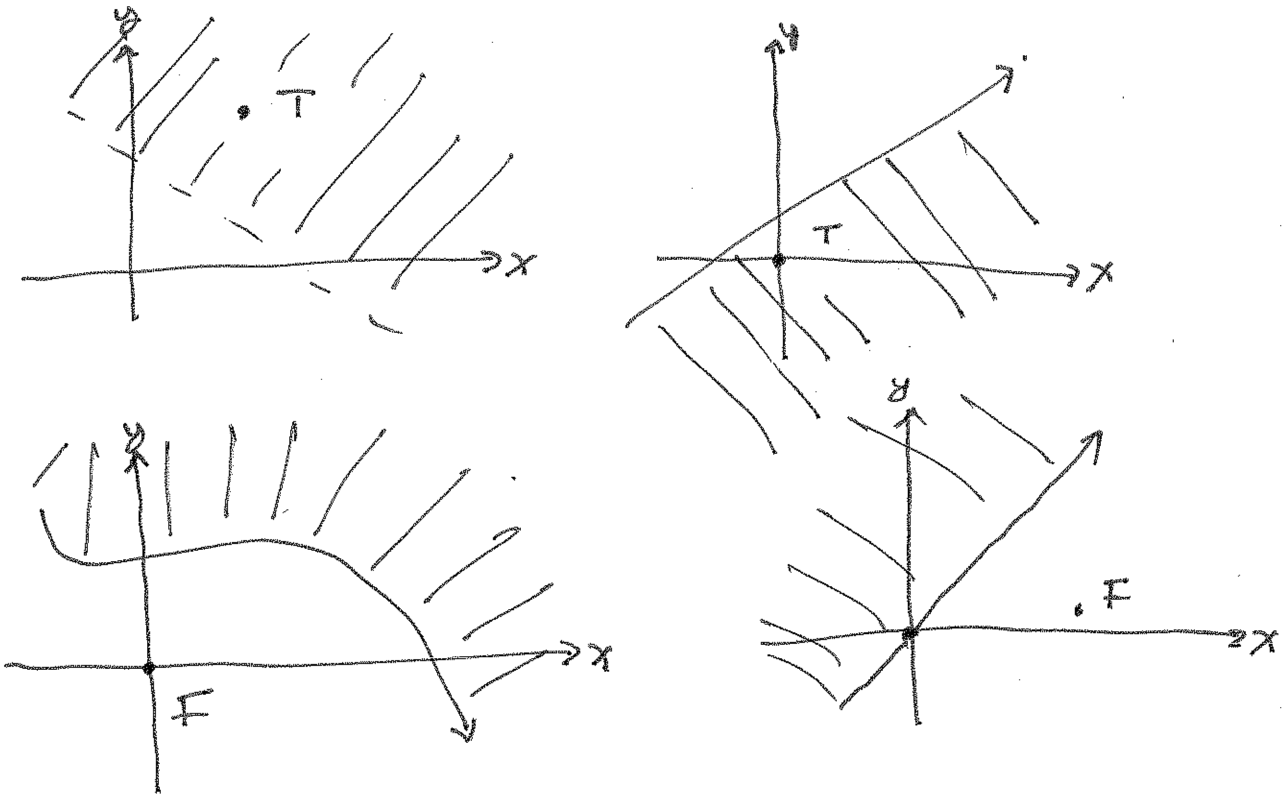
$$\left[\begin{array}{cc:c} 1 & 3 & 24 \\ 9 & 2 & 2 \end{array}\right] \quad R_2 - 9R_1 \rightarrow R_2$$

$$\left[\begin{array}{cc:c} 1 & 3 & 24 \\ 0 & -25 & -214 \end{array}\right] \quad -\frac{1}{25}R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc:c} 1 & 3 & 24 \\ 0 & 1 & \frac{214}{25} \end{array}\right] \quad R_1 - 3R_2 \rightarrow R_1$$

$$\left[\begin{array}{cc:c} 1 & 0 & -1.68 \\ 0 & 1 & \frac{214}{25} \end{array}\right] \qquad x = -1.68 \;;\quad y = \frac{214}{25} = 8.56$$

Which side should we shade?



Test a pt off the line

& shade each.

logs ... the rules.

- $a > 0$ & $a \neq 1$ $\quad \log_a a = 1$
- $\log(M \cdot N) = \log M + \log N$
- $\log\left(\frac{M}{N}\right) = \log M - \log N$
- $\log(M^x) = x \log M$
- $\log_b a = \frac{\ln a}{\ln b}$
- $y = b^x \iff \log_b y = x$

ex: if $\log_b(2) = u$ & $\log_b(3) = v$ express the following in terms of u & v.

(a) $\log_b\left(\frac{2}{3}\right) = \underbrace{\log_b(2)}_{u} - \underbrace{\log_b(3)}_{v}$

$= u - v$

(b) $\log_b(6) = \log_b(2 \cdot 3)$

$= \log_b(2) + \log_b(3)$

$= u + v$

(c) $\log_b\left(\frac{12}{b}\right) = \log_b(12) - \log_b(b)$

$= \log_b(2^2 \cdot 3) - 1$

$= 2\log_b(2) + \log_b(3) - 1$

$= 2u + v - 1.$

Log Rules.

- $\log(MN) = \log M + \log N$
- $\log\left(\frac{M}{N}\right) = \log M - \log N$
- $\log(M^x) = x \log M$.

ex: If $\log_b(3) = u$ & $\log_b(5) = v$, find

(a) $\log_b(15) = \log_b(3 \cdot 5)$

$= \log_b(3) + \log_b(5)$

$= u + v$

(b) $\log_b\left(\frac{25}{27}\right) = \log_b(25) - \log_b(27)$

$= \log_b(5^2) - \log_b(3^3)$

$= 2\log_b(5) - 3\log_b(3)$

$= 2v - 3u$.

Arithmetic & Geometric Sequences & Series

Arithmetic	Geometric	Sequences	Series
common difference	common ratio.	list	add

$a_n = a_1 + d(n-1)$ $\qquad$ $a_n = a_1 \cdot r^{n-1}$

$S_n = \frac{n}{2}(a_1 + a_n)$ $\qquad$ $S_n = \dfrac{a_1(r^{n}-1)}{r-1}$

S_n is the sum of the 1st n terms.

ex: Shawn starts w/ 3 marbles & his collection doubles each day. How long until he has 2072 marbles to lose.

$a_1 = 3$; $\quad r = 2$; $\quad a_n = 3 \cdot 2^{n-1}$

solve $3072 = 3 \cdot 2^{n-1}$

$\Rightarrow 1024 = 2^{n-1}$

$\Rightarrow \log(1024) = (n-1)\log(2)$

$\Rightarrow n-1 = \dfrac{\log(1024)}{\log(2)} = 10$

$\Rightarrow n = 11$

Sum of the arithmetic series.

$$\underbrace{6 + 14 + 22 + \dots}_{200 \text{ terms.}}$$

recall

$$S_{200} = \frac{n(a_1 + a_n)}{2}$$

where $n = 200$.

find a_{200} ....

$d = 8$

$a_1 = 6$

$\Rightarrow a_n = 6 + 8(n-1)$

$\Rightarrow a_{200} = 6 + 8(199) = 1598$

Hence $S_{200} = \dfrac{200(6 + 1598)}{2}$

$= 160{,}400$

Population modeling

$$P(t) = P_0 (1+r)^t$$

OR

$$P(t) = P_0 e^{kt}$$

P_0 = initial population.

ex: The pop. of Mathville is 31,415 today growing @ 1.41% annually....

model: $P(t) = 31415(1.0141)^t$

t	P
0	31415
1	31858

Geometric sequence to solve finance questions

→ TMV solver

→ Deb invests \$1000 @ 20% compounded annually for 10 yrs. How much does she have?

$a_1 = 1000$

$a_2 = 1000(1.2)$

$a_3 = 1000(1.2)^2$

$\vdots$

$a_n = 1000(1.2)^{n-1}$

Geometric formulas

$a_n = a_1 \cdot r^{n-1}$

$S_n = \dfrac{a_1(1-r^n)}{1-r}$

$a_{11} = 1000(1.2)^{10} = \$6191.74$

Deb has \$6191.74 after 10 yrs.

periodic vs. continuous compounding

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

vs.

$$A = Pe^{rt}$$